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Dynamic Nonlinear Hebbian Learning Constrained Optimization in Fuzzy Cognitive Maps: Application to a Process Control Benchmark

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ABSTRACT
Fuzzy Cognitive Maps (FCMs) are commonly used to model complex systems where variables interact through causal relationships and feedback. In practical control environments, some variables must remain within defined bounds to ensure safe and stable operation. However, many existing learning algorithms for FCMs do not consider the operational limits that often exist for important system outputs. This study addresses this limitation by proposing a Dynamic Nonlinear Hebbian Learning (D-NHL) algorithm that incorporates constraint awareness directly into the weight update process. The proposed method modifies the traditional NHL rule by introducing conditional updates based on whether selected output concepts satisfy predefined operational bounds. The algorithm was evaluated using a benchmark chemical process control problem involving the regulation of liquid height and specific gravity in a mixing tank. Results from the experiments show that both the conventional NHL algorithm and the proposed D-NHL method achieve convergence during learning. However, the D-NHL algorithm consistently guides the system toward equilibrium states that keep the critical output variables within the specified limits. Quantitative evaluation also shows a slight improvement in predictive performance, with the mean squared error (MSE) reduced from 0.072 to 0.069. These findings suggest that embedding constraint handling within the learning process improves the suitability of FCM models for control-oriented applications where operational limits must be maintained. This makes the approach relevant for areas such as industrial process monitoring and other decision-support environments. Future work will focus on applying the method to larger FCM structures and validating its performance using real-world datasets and operational systems.

Keywords:
Fuzzy Cognitive Maps,
Nonlinear Hebbian Learning,
Process Control,
Constrained Optimization,
Dynamic Systems,
Intelligent Control.

INTRODUCTION

Fuzzy Cognitive Maps (FCMs) were introduced by Kosko (1986), and are soft computing models that combine elements of neural networks and fuzzy logic to represent causal reasoning in complex systems where uncertainty, nonlinearity, and incomplete knowledge coexist. They are directed graphs whose nodes represent system concepts while the weighted edges represent causal influences between these concepts. The state of the system evolves iteratively through nonlinear transformations of weighted concept interactions. As cognitive models, they support both qualitative and quantitative reasoning, while allowing expert knowledge to be incorporated alongside data-driven learning (Karatzinis & Boutalis, 2025).

FCMs have been successfully applied over the years in domains including but not limited to decision support (Borrero-Domínguez & Escobar-Rodríguez, 2023), medical modelling (Apostolopoulos et al., 2024), environmental management (Papageorgiou & Kontogianni, 2012), and industrial systems (Boutalis et al., 2014). In control and supervisory environments, they are widely used because they provide interpretability as well as adaptive learning capabilities.

One of the major challenges in FCM research is learning the weight matrix so that the model converges to desirable equilibrium states. Although expert-defined weights are common, automated learning improves model accuracy and objectivity. Among the various learning schemes proposed though literature, Hebbian-based methods remain popular due to their simplicity and convergence properties (Osório et al., 2024). However, standard learning algorithms typically optimize global error without explicitly enforcing operational bounds on specific output concepts.

In domains such as industrial process control, safety systems, and regulatory environments, maintaining system outputs within prescribed bounds is a very important requirement that standard learning rules cannot always guarantee. Therefore, incorporating constraint-awareness into the learning process is necessary for practical deployment.

FCMs have been applied across several key sectors because they provide transparent causal reasoning while also integrating diverse sources of knowledge. A recent survey of FCMs in engineering applications systematically categorized over 80 studies across 15 sub-domains, including Control Systems, Decision Support Systems, Robotics, Reliability and Safety Systems, Energy Systems, Fault Detection and Diagnosis, Maritime Systems, Energy Economics among others (Karatzinis & Boutalis, 2025).

In participatory research, FCMs support multi-stakeholder modeling by representing diverse causal understandings and facilitating intervention planning without prioritizing purely numerical accuracy (Sarmiento et al., 2024). In medicine, FCMs have been used to model personalized phenomena, such as pain perception, by integrating expert input and simulating fixed-point behaviour that reflects complex biopsychosocial interactions (Farahani et al., 2025). These applied contexts highlight the versatility of FCMs but also expose limitations when learning mechanisms fail to enforce constraints that are inherent to real-world systems.

This paper proposes a Dynamic Nonlinear Hebbian Learning (D-NHL) algorithm that extends the classical NHL by embedding output-bound constraints directly into the update mechanism thereby ensuring feasible equilibrium states for output concepts. The proposed method is evaluated using a benchmark chemical mixing process control problem which involves the regulation of liquid height and specific gravity in a mixing tank.

Early FCM learning algorithms were adapted from artificial neural networks, particularly unsupervised Hebbian learning rules. Tools such as FCMpy provide extensive support for constructing, simulating, and learning FCMs using algorithms such as Nonlinear Hebbian, Active Hebbian, and evolutionary methods (Mkhitaryan et al., 2022).

Recent advances in learning mechanisms include neural network-enhanced FCM variants that compute system biases, offering improved interpretability and adjustment of concept influences (Sabahi & Stanfield, 2022). Gradient and optimization-based methods such as backpropagation applied to FCM models offer alternative weight update mechanisms, particularly in classification problems where predictive performance is essential (Quesada & Concepci, 2024). While these methods improve performance metrics and introduce hybrid dynamics, they generally lack mechanisms for enforcing hard constraints on output concept values, a significant gap for control applications.

Convergence behaviour is a major research concern in FCM dynamics. Stability of the reasoning process is defined as convergence to a fixed point, which depends on weight magnitudes and activation functions. Harmati et al. (2023) studied global stability properties in FCMs, establishing conditions under which FCM systems converge reliably. Revised cognitive mapping methodologies seek to address interpretability and dynamic behaviour but do not necessarily integrate constraints within the learning process (Nápoles et al., 2024). Existing theoretical work supports convergence under controlled weight conditions, but practical enforcement of operational bounds remains an open challenge.

Extensions of FCMs, including fuzzy grey cognitive maps (Salmeron, 2010), intuitionistic FCMs (Papageorgiou & Iakovidis, 2013), and dynamic fuzzy grey cognitive maps (Chen et al., 2021) enhance modeling under various types of uncertainty and hesitancy but primarily focus on representing uncertainty rather than enforcing output constraints. For example, Sovatzidi et al. (2025) proposed intuitionistic fuzzy cognitive maps to enhance interpretability in image classification contexts, capturing hesitancy along with membership relationships.

Hybrid learning methods usually blend Hebbian-based algorithms with population-based ones for robust weight training. Some of these approaches include the combination of Nonlinear Hebbian Learning and Genetic Algorithms (NHL-GA) by Natarajan et al. (2016), Data-Driven NHL and Extended Great Deluge Algorithm (NHL-EGDA) by Ren (2012). In these methods, NHL is used to initialize weights from experts or existing data, then population-based metaheuristics are used to refine for stability. This research builds on these foundations by introducing a constraint-aware adaptation within the Hebbian-based learning mechanism, to achieve a controlled convergence of output concepts.

METHODOLOGY

Extending Nonlinear Hebbian Learning Using Dynamic Weight Update

In the original NHL formulation by Papageorgiou et al. (2003), weight adaptation is achieved by maximising the criterion function in (1):

$$J = E[z^2] \qquad (1)$$

Where $z = f(y), y = W^T X$ and $f(\cdot)$ is a nonlinear activation function. The stochastic gradient update leads to the classical nonlinear Hebbian weight rule (2):

$\Delta w_{ji} = \eta_k A_j (A_i - A_j w_{ji})$ (2)

NHL therefore adapts weights based on the co-activation of pre-synaptic and post-synaptic concepts, which is governed by nonlinear activation shaping.

Motivation for extending NHL

While NHL improves structural learning in FCMs, it assumes that concept activations evolve freely under the influence of weighted interaction. In real systems however, some concepts (DOCs) must remain within bounded value intervals. Exceeding these intervals in such systems may represent unacceptable system states and the classical NHL has no mechanism to enforce such constraints. This implies that even though weight learning converges mathematically, it may be to operationally invalid equilibrium states.

Dynamic Constraint incorporation

The Dynamic NHL (D-NHL) algorithm introduces bounded activation dynamics as in (3) which are governed by user-defined minimum and maximum acceptable values:

$A_i^{Min} \leq A_i \leq A_i^{Max}$ (3)

Where

A_i^{Min} is the minimum acceptable value for concept A_i

A_i^{Max} is the maximum acceptable value for concept A_i

When the updated activation A_i violates a constraint, an adjustment term u is applied as follows (4):

$$u = \begin{cases} A_i^{Max} - A_i, & A_i > A_i^{Max}, \\ A_i^{Min} - A_i & A_i < A_i^{Min}, \\ 0, & otherwise. \end{cases} \quad (4)$$

This ensures that DOCs remain within valid operational ranges. Corrections occur before weight updates, and that learning does not propagate invalid concept activations.

Once the activation is dynamically corrected, the D-NHL weight update follows a modified nonlinear Hebbian form (5):

$W_{ji}^{k+1} = W_{ji}^{k} + \eta_k A_j (A_i - A_j W_{ji})$ (5)

The Proposed Dynamic Nonlinear Hebbian Learning Approach

At each iteration k, each concept C_i updates its activation using (6):

$$A_j^{k+1} = f\left(A_i^k + \sum_{\substack{j \neq i \\ j=i}}^{n} A_j^k W_{ji}^k\right) \quad (6)$$

Where:

$f(\cdot)$ is a sigmoidal activation ensuring bounded outputs and all nodes update synchronously.

For each DOC C_i, the activation is dynamically regulated as follows (7):

$A_i^{k+1} := A_i^{k+1} + u_i$ (7)

Where the adjustment u_i is defined in (7) to enforce (8):

$A_i^{Min} \leq A_i^{k+1} \leq A_i^{Max}$ (8)

The D-NHL algorithm checks for two convergence conditions (9) and (10) to be met before termination:

i. Stability condition: $A_i^{k+1} = A_i^k| \leq e, \forall i,$ (9)

ii. Constraint satisfaction: $A_i^{Min} \leq A_i^{k+1} \leq A_i^{Max}$, $\forall\, DOCs.$ (10)

If these conditions are not satisfied and the maximum number of iterations has not been hit, the algorithm continues.

The algorithm is presented as follows in Algorithm 1:

Algorithm 1: Dynamic NHL Algorithm

Step 1: Get the initial state activation vectors of A^0 and weight connection matrix W^0

Step 2: Specify the Min and Max values of Desired Output Concepts (A_P). P = 1 to *n*; *n* = total number of DOC

Step 3: If A_i is a DOC

For $i = 1$ to n

Calculate $A_j^{k+1} = f\left(A_i^k + \sum_{\substack{j \neq i \\ j=i}}^{n} A_j^k W_{ji}^k\right)$

Update the node according to the following steps:

If $A_i^{Max} < A_i$

Then $u = A_i^{Max} - A_i$

Else if $A_i^{Max} > A_i$

Then $u = A_i^{Max} + A_i$

Else $W_{ji}^k + \eta_k A_j (A_i - A_j W_{ji})$

Step 4: Check the convergence condition for all nodes of the FCM

4.1: $|A_i^{k+1} = A_i^k| \leq e$

4.2: For all DOCs

$A_i^{Min} \leq A_i \leq A_i^{Max}$

Step 5: Calculate the cost function

$$\sum_{i=1}^{n} (DOC)^2$$

Step 6:	If 4.1 and 4.2 are satisfied Then output the W_{ji} and A_i Stop Else if Maximum iterations reached Stop Else go to Step 3

Implementing The Process Control Problem

The process control problem is an optimisation problem commonly used as a benchmark to test Fuzzy Cognitive Map (FCM) algorithms. It is a chemical process control system that involves modelling and controlling a chemical mixing process within a single tank fluid regulated by valves.

The physical setup is defined by:

i. A central mixing Tank.

ii. Valve 1 and Valve 2 which supply two different liquids into the tank, where they are mixed and a chemical reaction takes place.

iii. A sensor in the tank that measures specific gravity of the liquid resulting from the chemical reaction

iv. Valve 3 controls the drainage of the mixed liquid from the tank.

The objective is to design a Fuzzy Cognitive Map capable of supervisory control over this process such that the controlled system must maintain specific gravity (G) variables as well as height of the liquid in the tank (H) within defined limits by adjusting the inputs as in (11):

$$G_{min} \leq G \leq G_{max};\ H_{min} \leq H \leq H_{max} \qquad (11)$$

The FCM model must determine the precise states (optimal weights) of Valve 1, Valve 2, and Valve 3 such that, given any initial state, the system converges to a fixed equilibrium point where these desired output concepts satisfy their predetermined bounds.

From the problem, a FCM with five concepts is constructed as follows:

Concept 1 (C_1): The amount of liquid in the tank measured by its height H. This is determined by the operational state of valves 1, 2 and 3.

Concept 2 (C_2): The state of valve 1 which could be closed, partially open or opened.

Concept 3 (C_3): The state of valve 2 which could be closed, partially open or opened.

Concept 4 (C_4): The state of valve 3 which could be closed, partially open or opened.

Concept 5(C_5): The specific gravity of the liquid in the tank.

Expert defined initial weights for the model which are the initial point for learning are presented in the weight matrix below and Fig 1:

$$W^{initial} = \begin{bmatrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ C_1 & 0 & -0.4 & -0.25 & 0 & 0.3 \\ C_2 & 0.36 & 0 & 0 & 0 & 0 \\ C_3 & 0.45 & 0 & 0 & 0 & 0 \\ C_4 & -0.9 & 0 & 0 & 0 & 0 \\ C_5 & 0 & 0.6 & 0 & 0.3 & 0 \end{bmatrix}$$

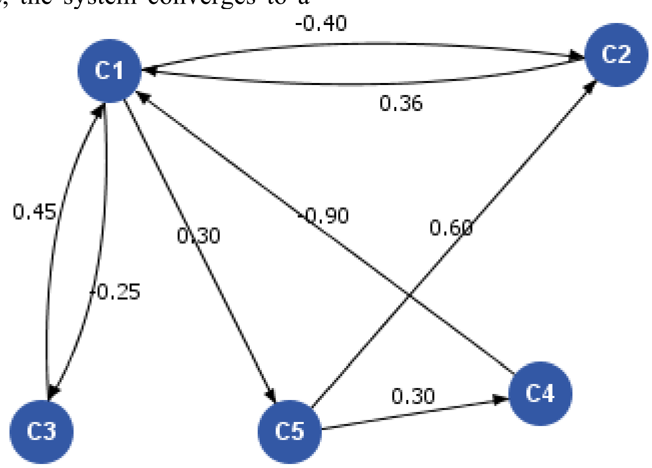

Figure 1: FCM for process control problem based on expert-defined weights

Scenario 1

The following learning parameters were used:

Learning rate = 0.0001

λ values = 1

e = 0.001

RESULTS AND DISCUSSION

Weight Matrices

The NHL algorithm produced the following learned weight matrix:

$W^{NHL} =$

$$\begin{bmatrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ C_1 & 0 & -0.316 & -0.182 & 0.063 & 0.346 \\ C_2 & 0.397 & 0 & 0.060 & 0.070 & 0.069 \\ C_3 & 0.480 & 0.061 & 0 & 0.062 & 0.061 \\ C_4 & -0.774 & 0.070 & 0.061 & 0 & 0.070 \\ C_5 & 0.062 & 0.627 & 0.061 & 0.349 & 0 \end{bmatrix}$$

The updated weight matrix for Dynamic NHL was:

$W^{D-NHL} =$

$$\begin{bmatrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ C_1 & 0 & -0.315 & -0.181 & 0.063 & 0.326 \\ C_2 & 0.397 & 0 & 0.061 & 0.070 & 0.040 \\ C_3 & 0.481 & 0.061 & 0 & 0.062 & 0.035 \\ C_4 & -0.773 & 0.070 & 0.061 & 0 & 0.041 \\ C_5 & 0.062 & 0.628 & 0.061 & 0.350 & 0 \end{bmatrix}$$

From the learned weights of both NHL and D-NHL it is clear that the algorithms were able to maintain stability and came to a convergence point. However, D-NHL was able to dynamically adjust weaker connections as can be seen in the improved weight values of specific gravity (C_5). The controlled weight adaptation of D-NHL prevents large oscillations which leads to better numeric stability and smaller steady-state error.

Final Concept Values

The final optimal values of the concepts for this scenario are presented as:

$A_{NHL} = [0.680 \quad 0.751 \quad 0.663 \quad 0.762 \quad 0.757]$

$A_{D-NHL} = [0.680 \quad 0.749 \quad 0.663 \quad 0.761 \quad 0.740]$

Scenario 2

Learning the following 3 concepts using the intervals below:

Concept 1: between 0.74 to 0.80 $0.74 \leq C_1 \leq 0.80$ $0.2 \leq C_4 \leq 0.3$ $0.68 \leq C_5 \leq 0.70$

Concept 4: between 0.2 to 0.3

Concept 5: between 0.68 to 0.70

Weight Matrices

There was no convergence of the NHL algorithm after 10,000 iterations. However, the updates weight matrix produced was:

$W^{NHL} =$

$$\begin{bmatrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ C_1 & 0 & 0.305 & 0.355 & 0.513 & 0.665 \\ C_2 & 0.674 & 0 & 0.493 & 0.524 & 0.524 \\ C_3 & 0.721 & 0.504 & 0 & 0.508 & 0.508 \\ C_4 & 0.772 & 0.522 & 0.495 & 0 & 0.526 \\ C_5 & 0.503 & 0.806 & 0.494 & 0.668 & 0 \end{bmatrix}$$

After 9046 iterations the updated weight matrix for Dynamic NHL was:

$W^{D-NHL} =$

$$\begin{bmatrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ C_1 & 0 & 0.146 & 0.216 & -0.406 & 0.328 \\ C_2 & 0.243 & 0 & 0.421 & -0.545 & 0.043 \\ C_3 & 0.357 & 0.433 & 0 & -0.477 & 0.038 \\ C_4 & -1.051 & 0.340 & 0.313 & 0 & 0.041 \\ C_5 & -0.178 & 0.799 & 0.402 & 0.026 & 0 \end{bmatrix}$$

Final concept values:

$A_{NHL} = [0.943 \quad 0.952 \quad 0.936 \quad 0.955 \quad 0.955]$

$A_{D-NHL} = [0.680 \quad 0.886 \quad 0.854 \quad 0.300 \quad 0.740]$

Scenario 3

Learning the following 5 concepts using the intervals below:

Concept 1 between 0.74 to 0.80 $0.74 \leq C_1 \leq 0.80$ $0.4 \leq C_2 \leq 0.5$ $0.4 \leq C_3 \leq 0.5$ $0.2 \leq C_4 \leq 0.3$ $0.68 \leq C_5 \leq 0.70$

Concept 2: between 0.4 to 0.5

Concept 3: between 0.4 to 0.5

Concept 4: between 0.2 to 0.3

Concept 5: between 0.68 to 0.70

Weight Matrixes

There was also no convergence of the NHL algorithm after 10,000 iterations. The updated weight matrix was:

$W^{NHL} =$

$$\begin{bmatrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ C_1 & 0 & 0.305 & 0.355 & 0.513 & 0.665 \\ C_2 & 0.674 & 0 & 0.493 & 0.524 & 0.524 \\ C_3 & 0.721 & 0.504 & 0 & 0.508 & 0.508 \\ C_4 & 0.772 & 0.522 & 0.495 & 0 & 0.526 \\ C_5 & 0.503 & 0.806 & 0.494 & 0.668 & 0 \end{bmatrix}$$

The updated weight matrix for Dynamic NHL was:

$W^{D-NHL} =$

$$\begin{bmatrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ C_1 & 0 & -0.902 & -0.414 & -0.497 & 0.340 \\ C_2 & 0.371 & 0 & -0.122 & -0.392 & 0.055 \\ C_3 & 0.457 & -0.250 & 0 & -0.341 & 0.480 \\ C_4 & -0.831 & -0.304 & -0.125 & 0 & 0.056 \\ C_5 & -0.011 & 0.495 & -0.141 & -0.054 & 0 \end{bmatrix}$$

Final concept values

$A_{NHL} = [0.943 \quad 0.952 \quad 0.936 \quad 0.955 \quad 0.955]$

$A_{D-NHL} = [0.680 \quad 0.500 \quad 0.500 \quad 0.420 \quad 0.740]$

A summary of the performance metrics for both NHL and Dynamic NHL are presented in Table 1.

Table 1: Performance Evaluation

Scenario	Algorithm	Constraint	No. of Iterations	MSE	RMSE
1	NHL	None	1341	0.072	0.268
	Dynamic NHL	None	1315	0.069	0.262
2	NHL	$0.74 \le C_1 \le 0.80$	Max	NC	NC
	Dynamic NHL	$0.2 \le C_4 \le 0.3$	9046	0.120	0.347
		$0.68 \le C_5 \le 0.70$			
3	NHL	$0.74 \le C_1 \le 0.80$	Max	NC	NC
	Dynamic NHL	$0.4 \le C_2 \le 0.5$	9064	0.096	0.311
		$0.4 \le C_3 \le 0.5$			
		$0.2 \le C_4 \le 0.3$			
		$0.68 \le C_5 \le 0.70$			

Max = 10,000 iterations; NC = No Convergence

In the first run (Scenario 1), there were no constraints imposed on any of the concepts; both NHL and D-NHL successfully converged to stable equilibrium points. However, D-NHL achieved better stability and smoother convergence, as indicated by a slightly lower Mean Square Error (0.069) and Root Mean Square Error (0.262) compared to NHL (MSE = 0.072, RMSE = 0.268). The final concept activations were close, showing that both algorithms maintained the desired system state within acceptable limits.

In the second scenario run, the goal was to maintain specific output intervals for three concepts (C_1, C_4, C_5), NHL failed to converge even after 10,000 iterations, exhibiting signs of weight explosion and numerical divergence. On the other hand, D-NHL achieved convergence in 9046 iterations, producing a sparse and balanced weight structure with stable concept activations. This performance demonstrates that D-NHL can adjust dynamically to nonlinear dependencies and evolving system states which is a major weakness of traditional NHL.

When all five concepts were simultaneously constrained, the standard NHL again failed to converge after 10,000 iterations, confirming its instability in controlling weights in a large system since by nature as it continues to learn, the weights keep growing. In contrast, D-NHL converged successfully with performance metrics (MSE = 0.096, RMSE = 0.311). These results show that D-NHL is able to effectively regularise weight updates, avoiding runaway amplification as the dynamic learning rate serves as a weight decay mechanism to stabilise learning. This is an improvement over the conventional NHL algorithm.

CONCLUSION

In industrial and process control settings, it is important to maintain specific operational ranges of system variables to ensure safe and stable operation. The proposed Dynamic NHL method in this study modified the traditional Nonlinear Hebbian Learning (NHL) rule by introducing conditional weight updates based on whether specified output concepts fall within predefined bounds. The approach was tested using a benchmark process control problem involving the regulation of liquid height and specific gravity in a mixing tank system. The results showed that both the conventional NHL algorithm and the proposed D-NHL method achieved convergence during learning. However, the D-NHL algorithm consistently drove the system toward equilibrium states that satisfied the specified operational limits. Performance evaluation using Mean Squared Error (MSE) and Root Mean Squared Error (RMSE) indicated that the proposed method also provided slightly better predictive stability than the standard NHL approach. At the same time, the algorithm maintained the simplicity and transparency associated with Hebbian learning, which are important strengths of FCM-based models. These results show that incorporating constraint awareness into the learning process can improve the practical usefulness of FCM models in situations where system variables must operate within safe or regulated ranges. By embedding constraint handling directly into the learning mechanism, the D-NHL algorithm makes FCMs more suitable for control-oriented applications such as industrial process monitoring, environmental systems management, and other decision-support environments where operational limits are important. The study also shows that the basic principles of Hebbian learning can be extended to address practical modeling challenges while preserving the interpretability and causal reasoning that make FCMs attractive for complex system analysis. Future research can focus on applying the D-NHL algorithm to larger FCM structures with more concepts and more complex causal interactions.

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