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Convergence Analysis and Numerical Solution of the BBM Equation using the Kamal-Adomian Decomposition Method

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ABSTRACT

The Benjamin-Bona-Mahony (BBM) equation is a nonlinear dispersive partial differential equation widely used to describe the propagation of long waves in fluid media and other physical systems. Due to the nonlinear nature of the equation, obtaining exact analytical solutions can be challenging. Consequently, semi-analytical techniques are often employed to obtain accurate approximations with reduced computational complexity. In this study, series solutions of the BBM equation are obtained using the Kamal-Adomian Decomposition Method (KADM). This approach combines the Kamal transform with the Adomian Decomposition Method, enabling the equation to be handled systematically by separating its linear and nonlinear components. The resulting solution is constructed iteratively as a convergent series without requiring linearization or discretization. To assess the efficiency of the method, several initial value problems are considered and the resulting solutions are compared with previously reported results in the literature. The method demonstrates rapid convergence, yielding highly accurate approximations with only a few terms of the series. Overall, the results demonstrate that KADM effectively captures the essential wave characteristics of the BBM equation, particularly the interaction between nonlinear steepening and dispersive effects. These findings indicate that the method is a reliable and efficient tool for solving nonlinear partial differential equations arising in applied mathematics and fluid dynamics.

Keywords:
Benjamin-Bona-Mahony equation,
Kamal-Adomian Decomposition Method,
Nonlinear PDEs,
Convergence Analysis,
Numerical Solution.

INTRODUCTION

The Benjamin-Bona-Mahony (BBM) equation is one of the well-known nonlinear partial differential equations used in wave modelling. It is frequently used to model the propagation of long waves in dispersive media such as shallow water. It was first presented by Benjamin et al. (1972) as a substitute for the Korteweg–de Vries (KdV) equation. It provides a more stable and physically consistent framework while addressing issues such as unbounded phase speeds and ill-posed initial–boundary problems.

The standard form of the BBM equation is:

$$\alpha_t + \alpha_c + \alpha\alpha_c - \alpha_{cct} = 0 \qquad (1)$$

Where $\alpha(c,t)$represents the wave profile, c represents the spatial variable and t represents the time variable. The BBM equation has a bounded dispersion relation, which makes it more appropriate for realistic wave modeling than the KdV equation (Zhao & Zhang, 2020).

Obtaining exact analytical solutions for nonlinear equations such as the BBM equation is often difficult due to the presence of nonlinear terms, and conventional direct approaches including the tanh and sine-cosine methods frequently depend on particular assumptions or boundary constraints, which restricts their broader applicability (Wazwaz, 2009; Akbar & Mahmood, 2016). To overcome these challenges, semi-analytical techniques, particularly the Adomian Decomposition Method (ADM), have been utilized. ADM produces rapidly converging series solutions without requiring linearization or the use of small parameters, and it has been successfully implemented for numerous nonlinear wave equations. (Adomian, 2013; Cherruault, 1995; Duan & Rach, 2012; Olubanwo et al., 2023).

Hybrid methods that combine integral transforms with decomposition techniques have also helped improve the efficiency of obtaining solutions. A notable example of this is the Kamal-Adomian Decomposition Method

(KADM), which effectively integrates the Kamal Transform with Adomian polynomials to iteratively disentangle and address both linear and nonlinear components (Pushpam & Kumar, 2019; Nisar, 2021).

In this study, KADM is applied to the BBM equation to construct rapidly convergent series solutions. The main objectives are to establish uniform convergence of the series, assess the accuracy of the approximations, and validate the results through comparison with existing analytical and semi-analytical solutions reported in recent literature. Through this work, the theoretical reliability and practical effectiveness of KADM for solving nonlinear dispersive wave models are further demonstrated.

MATERIALS AND METHODS

The Kamal Transform

The Kamal transform of a function $f(t)$ is defined as

$$K\{f(t)\} = R(v) = \int_0^\infty f(t)e^{\frac{-t}{v}}\,dt \tag{2}$$

Table 1: Kamal Transform of Frequently Encountered Functions

$f(t)$	$K\{f(t)\} = R(v)$
1	v
t	v^2
t^2	$2!\,v^3$
$t^n, n\epsilon N$	$n!\,v^{n+1}$
$t^n, n > -1$	$\Gamma(n+1)v^{n+1}$
e^{at}	$\frac{v}{1-av}$
$\sin at$	$\frac{av^2}{1+a^2v^2}$
$\cos at$	$\frac{v}{1+a^2v^2}$
$\sinh at$	$\frac{av^2}{1-a^2v^2}$
$\cosh at$	$\frac{v}{1-a^2v^2}$

Kamal Adomian Decomposition Method

Considering the Benjamin Bona Mahony Equation (BBM) of the form:

$$\alpha_t(c,t) = \alpha_{cct}(c,t) - \alpha(c,t)\alpha_c(c,t) - \alpha_c(c,t) \tag{3}$$

Subject to initial condition

$$\alpha(c,0) = g(c) \tag{4}$$

Take the Kamal transform of the equation (3)

$$K[\alpha_t(c,t)] = K[\alpha_{cct}(c,t) - \alpha(c,t)\alpha_c(c,t) - \alpha_c(c,t)] \tag{5}$$

Using the linearity and differential property of Kamal transform

$$\frac{1}{v}\alpha(c,v) - \alpha(c,0) = K[\alpha_{cct}(c,t) - \alpha(c,t)\alpha_c(c,t) - \alpha_c(c,t)]$$

$$\frac{1}{v}\alpha(c,v) = \alpha(c,0) + K[\alpha_{cct}(c,t)] - K[\alpha(c,t)\alpha_c(c,t)] - K[\alpha_c(c,t)] \tag{6}$$

Substitute the initial condition in equation (4) to equation (6)

$$\frac{1}{v}\alpha(c,v) = g(c) + K[\alpha_{cct}(c,t)] - K[\alpha(c,t)\alpha_c(c,t)] - K[\alpha_c(c,t)]$$

$$\alpha(c,v) = vg(c) + vK[\alpha_{cct}(c,t) - \alpha(c,t)\alpha_c(c,t) - \alpha_c(c,t)] \tag{7}$$

Take the inverse Kamal transform of equation (7)

$$\alpha(c,t) = K^{-1}[vg(c)] + K^{-1}[vK[\alpha_{cct}(c,t) - \alpha(c,t)\alpha_c(c,t) - \alpha_c(c,t)] \tag{8}$$

The series solution is given as:

$$\alpha(c,t) = \sum_{n=0}^{\infty} \alpha_n(c,t) \tag{9}$$

The nonlinear term is decomposed as:

$$\alpha\alpha_c = \sum_{n=0}^{\infty} A_n \tag{10}$$

Where A_n are Adomian Polynomials defined based on $\alpha$where $n = 0,1,2,\ldots$

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left(N\sum_{k=0}^{\infty}\lambda^k\alpha_k\right)_{\lambda=0}$$

Substituting equation (9) and (10) into equation (8)

$$\sum_{n=0}^{\infty}\alpha_n(c,t) = K^{-1}[vg(c)] + K^{-1}[vK[\alpha_{cct}(c,t) - \sum_{n=0}^{\infty}A_n - \alpha_c(c,t)] \tag{11}$$

The recursive relation is obtained by comparing the two sides of equation (11)

$$\alpha_0 = K^{-1}[vg(c)] = g(c) \tag{12}$$

$$\alpha_1 = K^{-1}[vK[\alpha_{0cct}(c,t) - A_0 - \alpha_{0c}(c,t)]$$

$$\alpha_1 = K^{-1}[vK[\alpha_{0cct}(c,t) - (\alpha_0\alpha_{0c}) - \alpha_{0c}(c,t) \tag{13}$$

$$\alpha_2 = K^{-1}[vK\{\alpha_{1cct}(c,t) - A_1 - \alpha_{1c}(c,t)]$$

$$\alpha_2 = K^{-1}[vK(\alpha_{1cct}(c,t) - (\alpha_1\alpha_{0c} + \alpha_0\alpha_{1c}] - \alpha_{1c}(c,t)] \tag{14}$$

$$\alpha_3 = K^{-1}[vK\,[\alpha_{2cct}(c,t) - A_2 - \alpha_{2c}(c,t)]$$

$$\alpha_3 = K^{-1}[vK(\alpha_{2cct}(c,t) - (\alpha_2\alpha_{0c} + \alpha_1\alpha_{1c} + \alpha_0\alpha_{2c})] \tag{15}$$

$$..\sum_{n=0}^{\infty}\alpha_n(c,t) = K^{-1}[vK[\alpha_{cct}(c,t) - \sum_{n=0}^{\infty}A_n - \alpha_c(c,t)] \tag{16}$$

Convergence Analysis of the Kamal–Adomian Decomposition Method

Before presenting the numerical simulations, the convergence and uniqueness of the series solution generated by the Kamal–Adomian Decomposition Method are established.

We consider the initial value problem

$\alpha_t + \alpha_c + \alpha\alpha_c - \alpha_{cct} = 0,$ (17)

$(c,t) \in \Omega \times [0,T],\qquad \alpha(c,t) = g(c)$

Where $\Omega \subset R$ is bounded and $T > 0$

Functional Framework

Let $X = C^1(\Omega \times [0,T])$

Equipped with the norm

$\|\alpha\|_X = \sup_{(c,t)\,\in\,\Omega\times[0,T]}(|\alpha(c,t)| + |\alpha_c(c,t)|).$ (18)

Then $(X, \|.\|_X)$ is a Banach Space.

Define the operators

$L(\alpha) = \alpha_t,\quad R(\alpha) = \alpha_c - \alpha_{cct},\quad N(\alpha) = \alpha\alpha_c$ (19)

The inverse operator of L is

$L^{-1}(f)(c,t) = \int_0^t f(c,s)ds.$

ApplyingL^{-1}, the BBM equation becomes the integral equation

$\alpha(c,t) = g(c) + \int_0^t[-R(\alpha)(c,s) - N(\alpha)(c,s)]ds.$ ` (20)

Define the operator $T: X \to X$ by

$(T\alpha)(c,t) = g(c) + \int_0^t[-R(\alpha)(c,s) - N(\alpha)(c,s)]ds.$ (21)

Theorem (Local Convergence and uniqueness of KADM)

Assume $g \in C^1(\Omega)$.

Let $B_M = \{\alpha \in X: \|\alpha\|_X \le M\}$ be a closed ball in X.

Then there exists $T > 0$ sufficiently small such that:

i. $T(B_M) \subset B_M$
ii. T Is a contraction onB_M.

Consequently, the integral admits a unique solution in B_M, and the Kamal–Adomian series

$\alpha = \sum_{n=0}^{\infty}\alpha_n$ (22)

Converges uniformly on $\Omega \times [0,T]$ to the unique solution.

Proof

For$f \in X$, $\|L^{-1}f\|_C \le T\|f\|_C$.

Thus the operator norm satisfies$\|L^{-1}f\|_X \le T$.

For $\alpha, v \in B_M$, $N(\alpha) - N(v) = (\alpha - v)\alpha_c + v(\alpha_c - v_c)$.

Using $\|\alpha\|_C, \|v\|_C \le M$,

$\|N(\alpha) - N(v)\|_C \le 2M\|\alpha - v\|_C.$ (23)

Hence, N is locally Lipschitz onB_M?

Since R is linear in α, there exists $P_R > 0$ such that

$\|R(\alpha) - R(v)\|_C \le P_R\|\alpha - v\|_C.$ (24)

For $\alpha, v \in B_M$, $\|T\alpha - Tv\|_C \le T(P_R + 2M)\|\alpha - v\|_C$.

Let$K = T(P_R + 2M)$.

Choosing $T > 0$ sufficiently small so that $K < 1$, we obtain

$\|T\alpha - Tv\|_C \le K\|\alpha - v\|_C.$ (25)

Thus T is a contraction.

By the Banach fixed-point theorem, T admits a unique fixed point$\alpha \in B_M$. The KADM construction corresponds to the successive Picard iterates of$T$, hence

$\alpha = \sum_{n=0}^{\infty}\alpha_n$

Converges uniformly to the unique solution of the BBM equation in$\Omega \times [0,T]$.

The KADM series converges locally and uniquely.

RESULTS AND DISCUSSION

Example 1. Consider the non-linear BBM equation of the form: (Olubanwo et.al, 2023)

$\alpha_t + \alpha_c + \alpha\alpha_c - \alpha_{cct} = 0$ (26)

Subject to the initial condition

$\alpha(c,0) = e^c$ (27)

Take the Kamal transform of equation (26)

$K[\alpha_t] = K[\alpha_{cct} - \alpha_c - \alpha\alpha_c]$

$\frac{1}{v}\alpha(c,v) - \alpha(c,0) = K[\alpha_{cct} - \alpha_c - \alpha\alpha_c]$

Substituting $\alpha(c,0) = e^c$

$\frac{1}{v}\alpha(c,v) = e^c + K[\alpha_{cct} - \alpha_c - \alpha\alpha_c]$

Multiply both sides by v

$\alpha(c,v) = ve^c + vK[\alpha_{cct} - \alpha_c - \alpha\alpha_c]$ (28)

Apply the inverse Kamal Transform

$\alpha(c,t) = K^{-1}[ve^c] + K^{-1}[vK\ (\alpha_{cct} - \alpha_c - \alpha\alpha_c)]$

$\alpha(c,t) = g(c) + K^{-1}[vK\ (\alpha_{cct} - \alpha_c - \alpha\alpha_c)]$ (29)

Where $g(c) = \alpha(c,0) = \alpha_0(c,t) = e^c$

Applying Adomian decomposition,

$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}(N\sum_{k=0}^{\infty}\lambda^k\alpha_k)_{\lambda=0}$

$A_0 = \alpha_0\alpha_{0c}$

$A_1 = \alpha_1\alpha_{0c} + \alpha_o\alpha_{1c}$

$A_2 = \alpha_2\alpha_{0c} + \alpha_1\alpha_{1c} + \alpha_0\alpha_{2c}$

Recursive relation is given as:

$\alpha_{n+1} = K^{-1}[vK(\alpha_{ncct} - \alpha_{nc} - A_n)]$ (30)

When$n = 0$,

$\alpha_1 = K^{-1}[vK(\alpha_{0cct} - \alpha_{0c} - A_0)]$

$= K^{-1}[vK(\alpha_{0cct} - \alpha_{0c} - \alpha_0\alpha_{0c}]$

$= -K^{-1}[vK(e^c + e^{2c})]$

$= -K^{-1}[v^2(e^c + e^{2c})]$

$= -(e^c + e^{2c})t$ (31)

When$n = 1$,

$\alpha_2 = K^{-1}[vK(\alpha_{1cct} - \alpha_{1c} - A_1)]$

$= K^{-1}[vK(\alpha_{1cct} - \alpha_{1c} - (\alpha_1\alpha_{0c} + \alpha_0\alpha_{1c})]$

$= K^{-1}[vK(-e^c - 4e^{2c} + e^ct + 2e^{2c}t - (-e^{2c}t - e^{3c}t - e^{2c}t - 2e^{3c}t))]$

$= K^{-1}[vK(-(e^c + 4e^{2c}) + (e^c + 4e^{2c} + e^{3c})t)$

$= K^{-1}[-(e^c + 4e^{2c})v^2 + (e^c + 4e^{2c} + 3e^{3c})v^3]$

$= -(e^c + 4e^{2c})t + \left(\frac{e^c+4e^{2c}+3e^{3c}}{2!}\right)t^2$ (32)

When$n = 3$,

$\alpha_3 = K^{-1}[vK(\alpha_{2cct} - \alpha_{2c} - A_2]$

$= K^{-1}\Big(vK\Big(-e^c - 16e^{2c} + e^ct + 16e^{2c}t + 27e^{3c}t + e^ct + 8e^{2c}t - \frac{e^c}{2}t^2 - 4e^{2c}t^2 - \frac{9}{2}e^{3c}t^2 + e^{2c}t +$

$4e^{3c}t - \frac{1}{2}e^{2c}t^2 - 2e^{3c}t^2 - \frac{3}{2}e^{4c}t^2 - e^{2c}t^2 -$
$3e^{3c}t^2 - 2e^{4c}t^2 + e^{2c}t + 8e^{3c}t - \frac{1}{2}e^{2c}t^2 - 4e^{3c}t -$
$\frac{9}{2}e^{4c}t^2\Big)\Big)$
$= K^{-1}\Big(vK\Big(\big(-(e^c + 16e^{2c}) + (2e^c + 26e^{2c} +$
$39e^{3c})t - \frac{1}{2!}(e^c + 3e^{2c} + 27e^{3c} + 4e^{4c})t^2\Big)\Big)$
$= -(e^c + 16e^{2c})t + \frac{(2e^c+26e^{2c}+39e^{3c})}{2!}t^2 -$
$\frac{(e^c+3e^{2c}+27e^{3c}+4e^{4c})}{3!}t^3 \qquad (33)$
$\alpha(c,t) = \alpha_0 + \alpha_1 + \alpha_2 + \alpha_3$
$\therefore \alpha(c,t) = e^c - (e^c + e^{2c})t - (e^c + 4e^{2c})t +$
$\frac{(e^c+4e^{2c}+3e^{2c})}{2!}t^2 - (e^c + 16e^{2c})t +$
$\frac{(2e^c+26e^{2c}+39e^{3c})}{2!}t^2 - \frac{(e^c+3e^{2c}+27e^{3c}+4e^{4c})}{3!}t^3 \quad (34)$

Equation (34) which is the resulting exact solution of equation (26) which is similar to the result obtained in (Olubanwo et.al, 2023)

Example 2: Consider the Homogeneous Benjamin–Bona Mahony Equation of the form: (Olubanwo et.al, 2023)
$\alpha_t = \alpha_{cct} - \alpha\alpha_c - \alpha_c \qquad (35)$
Subject to the initial condition
$\alpha(c,0) = \text{sech}^2(\frac{c}{4}) \qquad (36)$
Take the Kamal transform of the equation
$\frac{1}{v}\alpha(c,v) - \alpha(c,0) = K[\alpha_{cct} - \alpha\alpha_c - \alpha_c]$
$\frac{1}{v}\alpha(c,v) - \text{sech}^2(\frac{c}{4}) = K[\alpha_{cct} - \alpha\alpha_c - \alpha_c]$
$\frac{1}{v}\alpha(c,v) = \text{sech}^2(\frac{y}{4}) + K[\alpha_{cct} - \alpha\alpha_c - \alpha_c]$
Multiply through by v
$\alpha(c,v) = vsech^2\left(\frac{c}{4}\right) + vK\ [\alpha_{cct} - \alpha\alpha_c - \alpha_c]$
Apply the inverse Kamal Transform
$\alpha(c,t) = K^{-1}\left[vsech^2\frac{c}{4}\right] + K^{-1}[vK(\alpha_{cct} - \alpha\alpha_c - \alpha_c)]$
$\alpha(c,t) = g(c) + K^{-1}[vK(\alpha_{cct} - \alpha_c - \alpha\alpha_c)$
Applying Adomian polynomial where $g(c) = \alpha(c,0) = \text{sech}^2(\frac{c}{4})$
$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}(N\sum_{k=0}^{\infty}\lambda^k\alpha_k)_{\lambda=0}$
$A_0 = \alpha_0\alpha_{0c}$
$A_1 = \alpha_1\alpha_{oc} + \alpha_0\alpha_{1c}$
$A_2 = \alpha_2\alpha_{0c} + \alpha_1\alpha_{1c} + \alpha_0\alpha_{2c}$
Recursive relation is given as
$\alpha_{n+1} = K^{-1}\big(vK(\alpha_{ncct} - \alpha_{nc} - A_n)\big)$
When $n = 0$
$\alpha_1 = K^{-1}\big(vK\ (\alpha_{0cct} - \alpha_{oc} - A_0)\big)$
$\quad = K^{-1}\Big(vK\ (\alpha_{0cct} - \alpha_{0y} - (\alpha_0\alpha_{oc})\Big)$
Where $A_0 = \alpha_0\alpha_{0c}$
$A_0 = \text{sech}^2\frac{c}{4} \times -\frac{1}{2}\text{sech}^2\frac{c}{4}tanh\frac{c}{4}$
$A_0 = -\frac{1}{2}\text{sech}^4\frac{c}{4}tanh\frac{c}{4}$

$\alpha_1 = K^{-1}\Big(vK\Big(0 + \frac{1}{2}\text{sech}^2\left(\frac{c}{4}\right)\tanh\left(\frac{c}{4}\right) +$
$\frac{1}{2}\text{sech}^4\left(\frac{c}{4}\right)\tanh\left(\frac{c}{4}\right)\Big)\Big)$
$\quad = K^{-1}\Big(vK\ (\frac{1}{2}\text{sech}^2(\frac{c}{4})\tanh\ (\frac{c}{4})(1 + \text{sech}^2(\frac{c}{4}))\ \Big)$
$\quad = K^{-1}\Big(\left(\frac{1}{2}\right)sech^2\left(\frac{c}{4}\right)\tanh(\frac{c}{4})(1 + \text{sech}^2\left(\frac{c}{4}\right)\Big)v^2$
$\quad = \frac{1}{2}\text{sech}^2(\frac{c}{4})\tanh\ (\frac{c}{4})(1 + \text{sech}^2(\frac{c}{4}))t \qquad (37)$
When $n = 1$
$\alpha_2 = K^{-1}[vK(\alpha_{1cct} - \alpha_{1c} - A_1)]$
$\quad = K^{-1}\big[vK\big(\alpha_{1cct} - \alpha_{1c} - (\alpha_1\alpha_{0c} + \alpha_o\alpha_{1c})\big)\big]$
Where $A_1 = \alpha_1\alpha_{0c} + \alpha_0\alpha_{1c}$
$A_1 = \Big(\frac{1}{2}sech^2\frac{c}{4}tanh\frac{c}{4} + \frac{1}{2}sech^4\frac{c}{4}tanh\frac{c}{4})t) \times$
$(-\frac{1}{2}\text{sech}^2\frac{c}{4}tanh\frac{c}{4}\Big) + \Big(\text{sech}^2\frac{c}{4}\Big) \times \Big(\text{sech}^2\frac{c}{4}\Big)$
$\alpha_2 = K^{-1}\Big[vK\Big(\frac{1}{2}\text{sech}^2(\frac{c}{4})\Big)\tanh^2\frac{c}{4} - \frac{1}{8}sech^6\frac{c}{4} -$
$\frac{3}{16}\text{sech}^6\ tanh\frac{c}{4} + \frac{1}{8}\text{sech}^2\frac{c}{4}\ tanh\frac{c}{4} + \frac{1}{4}\text{sech}^4\frac{c}{4}tanh\frac{c}{4} +$
$(\frac{1}{2}\text{sech}^4\frac{c}{4}\tanh\frac{c}{4} - \frac{1}{8}sech^6\frac{c}{4} - \frac{1}{4}\text{sech}^2\frac{c}{4}\tanh^2\frac{c}{4} +$
$\frac{1}{8}\text{sech}^4\frac{c}{4})\text{t} - ((-\frac{1}{4})\text{sech}^4\frac{c}{4}\tanh^2\frac{c}{4} -$
$\frac{1}{4}\text{sech}^6\frac{c}{4}\tanh^2\frac{c}{4} - \frac{1}{2}\text{sech}^6\frac{c}{4}tanh\frac{c}{4} + \frac{1}{8}sech^8\frac{c}{4} -$
$\frac{1}{4}\text{sech}^4\frac{c}{4}\tanh^2\frac{c}{4} + \frac{1}{8}sech^6\frac{c}{4})t\ \Big]$
Through simplification and using of double angle we arrive at
$\alpha_2 = \frac{1}{256} + sech^8(-104t + 23tcosh\frac{c}{2}) + \Big(\frac{1}{256} +$
$sech^8\Big(16 + \cosh c + tcosh\left(\frac{3c}{2}\right)\Big)\Big) +$
$\Big(\frac{1}{256} + \text{sech}^8(-107sinh\frac{c}{2} + 8\sinh c + \sinh\frac{3c}{2})\Big)$
(38)
Thus the result $\alpha(c,t)$ is expressed in the form
$\alpha(c,t) = \alpha_0(c,t) + \alpha_1(c,t) + \alpha_2(c,t) + \cdots$

$\alpha(c,t) = \text{sech}^2\frac{c}{4} + (\frac{1}{2}sech^8\frac{c}{4}tanh\frac{c}{4} + \frac{1}{2}\text{sech}^2\frac{c}{4}tanh\frac{c}{4})t +$
$\Big(\frac{1}{256} + sech^8(-104t + 23tcosh\frac{c}{2})\Big) +$
$(\frac{1}{256} + sech^8(16tcosh(c) + tcosh\frac{3c}{2})) +$
$(\frac{1}{256} + sech^8(-107sinh\frac{c}{2})) + 8\sinh + \sinh\frac{3c}{2})) \quad (39)$
Equation (39) which is the resulting exact solution of equation (33) which is similar to the result obtained in (Olubanwo et.al, 2023)

Example 3: Consider the non-linear BBM equation of the form :(Olubanwo et.al, 2023)
$\alpha_t(c,t) = \alpha_{cct}(c,t) - \alpha\alpha_c(c,t) - \alpha_c(c,t) \quad (40)$
With initial condition:
$\alpha(c,0) = c^2 \qquad (41)$
Take the Kamal transform of the equation
$K[\alpha(c,t)] = K[\alpha_{cct}(c,t) - \alpha\alpha_c(c,t) - \alpha_c(c,t)]$
$\frac{1}{v}\alpha(c,v) - \alpha(c,0) = K[\alpha_{cct} - \alpha\alpha_c - \alpha_c]$
Substituting $\alpha(c,0) = c^2$

$\frac{1}{v}\alpha(c,v) = c^2 + K[\alpha_{cct} - \alpha\alpha_c - \alpha_c]$ (42)

Multiplying all through by v

$\alpha(c,v) = vc^2 + vK\,[\alpha_{cct} - \alpha\alpha_c - \alpha_c]$

Take the inverse Kamal Transform

$\alpha(c,t) = K^{-1}[vc^2] + K^{-1}\big[vK[\alpha_{cct} - \alpha\alpha_c - \alpha_c]\big]$

$\alpha(c,t) = g(c) + K^{-1}[vK(\alpha_{cct} - \alpha_c - \alpha\alpha_c)$ (43)

Where $g(c) = \alpha(c,0) = \alpha_0(c,t) = c^2$

Applying Adomian polynomial

$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}(N\sum_{k=0}^{\infty}\lambda^k\alpha_k)_{\lambda=0}$

$A_0 = \alpha_0\alpha_{0c}$

$A_1 = \alpha_1\alpha_{oc} + \alpha_0\alpha_{1c}$

$A_2 = \alpha_2\alpha_{0c} + \alpha_1\alpha_{1c} + \alpha_0\alpha_{2c}$

Recursive relation is given as:

$\alpha_{n+1} = K^{-1}\big(vK(\alpha_{ncct} - \alpha_{nc} - A_n)\big)$

When $n = 0$

$\alpha_1 = K^{-1}\big(vK\;(\alpha_{0cct} - \alpha_{oc} - A_0)\big)$

$\alpha_1 = K^{-1}\big(vK\;\big(\alpha_{0cct} - \alpha_{0c} - (\alpha_0\alpha_{oc}\big)\big)$

Where $A_0 = \alpha_0\alpha_{0c}$

$A_0 = c^2 \times 2c$

$A_0 = 2c^3$

$\alpha_1 = K^{-1}[vK(-2c^3 - 2c)$

$\quad = -K^{-1}[v^2(2c^3 + 2c)$

$\quad = -(2c^3 + 2c)t$ (44)

When$n = 1$,

$\alpha_2 = K^{-1}\big(vK\;(\alpha_{1cct} - \alpha_{1c} - A_1)\big)$

Where $A_1 = \alpha_1\alpha_{0c} + \alpha_0\alpha_{1c}$

$\alpha_2 = K^{-1}\big[vK[\alpha_{1cct} - (\alpha_1\alpha_{0c} + \alpha_0\alpha_{1c}) - \alpha_{1c}]\big]$

$= K^{-1}[vK[-12c - (-(2c^3 + 2c)t).2c + (c^2.(-(6c^2 + 2)t) - (-(6c^2 + 2)t)$

$\quad = -K^{-1}[v^2[-12c - \left(\frac{10c^4+6c^2+1}{s^2}\right)$

$\quad = [12ct + (5c^4 + 6c^2 + 1\,)t^2)$ (45)

$\alpha(c,t) = \alpha_o + \alpha_1 + \alpha_2$

$\alpha(c,t) = c^2 - (2c^3 + 2c)t + (12ct + (5c^4 + 6c^2 + 1)t^2)$

$\therefore \alpha(c,t) = c^2 + 12ct - (2c^3 + 2c)t + (5c^4 + 6c^2 + 1)t^2$ (46)

Equation (46) which is the resulting exact solution of equation (45) which is similar to the result obtained in (Olubanwo et.al, 2023)

Discussion

The application of the Kamal–Adomian Decomposition Method (KADM) to the Benjamin–Bona–Mahony equation produced series solutions that closely agree with previously reported results in the literature. This agreement confirms the accuracy and reliability of the proposed approach. By combining the Kamal transform with the systematic construction of Adomian polynomials, the nonlinear components of the equation can be handled effectively, which simplifies the solution procedure.

The results obtained from the numerical examples demonstrate that the series generated by KADM converges rapidly. In most cases, only a few terms of the decomposition series are required to obtain highly accurate approximations. This rapid convergence significantly reduces computational effort while maintaining solution accuracy.

Another important observation is that the method successfully captures the essential physical behavior described by the BBM equation, particularly the balance between nonlinear wave steepening and dispersive effects. Unlike many traditional analytical techniques, the present method does not require linearization, perturbation parameters, or discretization schemes.

Because of its simplicity, stability, and strong convergence properties, the Kamal–Adomian Decomposition Method provides an effective alternative for solving nonlinear dispersive wave equations. Furthermore, the general framework of the method makes it suitable for extension to other nonlinear partial differential equations encountered in applied mathematics, fluid dynamics, and engineering applications.

CONCLUSION

This study demonstrates that the Kamal–Adomian Decomposition Method (KADM) provides an efficient and reliable technique for solving the Benjamin–Bona–Mahony equation. The method generates rapidly convergent series solutions that accurately describe the key dynamics of the model while maintaining relatively low computational cost. By treating nonlinear terms directly without linearization, the method preserves the original structure of the equation and enhances analytical clarity.

The results further indicate that KADM is not limited to the BBM equation alone. Due to its flexibility and straightforward implementation, the method can be extended to a wider class of nonlinear dispersive equations. Consequently, KADM represents a promising analytical tool for future research in applied mathematics, fluid dynamics, and related scientific fields.

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