Nigerian Journal of Physics (NJP)
ISSN online: 3027-0936
ISSN print: 1595-0611
DOI: https://doi.org/10.62292/njp.v35i1.2026.482
Volume 35(1), March 2026

Numerical Solution of the Modified Radioactive Decay Rate Equation Using the Runge-Kutta Fourth Order Method

*[1]Otor, Daniel Abi, [2]Agaku, Raymond Msughter, [2]Bem, Timothy Terngu, [3]Peter, Ikpe Adoga, [4]Nyiyongo, Sesugh Emmanuel and [1]Noah, Ojodale Meshach

[1]Department of Physics, College of Physical Sciences, Joseph Sarwuan Tarka University, Markurdi, Nigeria.
[2]Department of Physics, Faculty of Science, Rev. Fr. Moses, Orshio Adasu University, Makurdi, Nigeria.
[3]ICT Directorate, National Open University of Nigeria Headquarters, Abuja, Nigeria.
[4]Department of Physics, Faculty of Physical Sciences, University of Mkar-Gboko, Nigeria.

*Corresponding Author's Email: otor.daniel@uam.edu.ng

ABSTRACT
The standard radioactive decay equation describes exponential decay but neglects possible production mechanisms that may arise in complex systems. This study investigates a modified radioactive decay rate equation that incorporates a quadratic production term, allowing for the modeling of coupled decay–production dynamics. The governing nonlinear ordinary differential equation was solved analytically for limiting cases and numerically using the fourth-order Runge–Kutta (RK4) method. Numerical simulations were performed for varying values of the production parameter β, while keeping the decay constant fixed, allowing for a direct comparison between standard decay and modified decay behaviors. The results show close agreement between analytical and numerical solutions at low β values, confirming the accuracy of the RK4 method. As β increases, deviations from simple exponential decay become significant, with the emergence of steady-state solutions where production balances decay. These steady states occur when the production term becomes comparable to the linear decay term. The study demonstrates that the modified model captures decay dynamics not represented by the standard equation and highlights the sensitivity of system behaviors to the quadratic production parameter. The findings confirm the suitability of the RK4 method for solving nonlinear radioactive decay models and provide a framework for extending the analysis to more complex decay systems involving spatial dependence or coupled nuclide chains.

Keywords:
Modified radioactive decay,
Nonlinear decay model,
Runge–Kutta fourth order,
Decay production dynamics,
Numerical simulation.

INTRODUCTION

Radioactivity is a fundamental nuclear process through which unstable atomic nuclei transform into more stable configurations. This transformation occurs through several decay mechanisms, including alpha decay, beta decay, gamma emission, and spontaneous fission, often accompanied by the release of high-energy particles or photons. Radioactive decay plays a critical role in nuclear physics, astrophysics, medical physics, radiometric dating, and reactor technology, where accurate modeling of decay processes is essential for both theoretical understanding and practical applications (Mumpower et al., 2016).

The Standard radioactive decay is the spontaneous, random process by which unstable atomic nuclei lose energy by emitting radiation (alpha particles, beta particles, or gamma rays) to achieve a more stable state. This process is characterized by first-order kinetics, meaning the rate of decay is proportional to the number of atoms present, resulting in a constant half-life for any given isotope (Mumpower et al., 2016). While generally reliable, radioactive decay has several limitations in theory and application: randomness at atomic level, time-range limitations (Dating), environmental assumptions (Carbon-14), environmental interference, measurement limits and system closure (Dating). The motivation for introducing nonlinear modifications to the standard, linear radioactive decay rate equation $\left(\frac{\mathrm{dN}}{\mathrm{dt}} = -\lambda N\right)$ stems from the need to model complex physical phenomena that deviate from simple exponential decay, such as high-intensity, non-stationary conditions, or coupling between multiple decay processes. The Fourth Order Runge-Kutta (RK4) method is utilized to solve these modified equations due to its high accuracy, stability, and capability to handle non-linear ordinary differential equations (ODEs) where analytical solutions are unavailable.

The behavior of radioactive nuclides is commonly characterized using the decay constant λ or the half-life$T_{1/2}$, which provides a measure of nuclear instability. The half-life is defined as the time required for half of the radioactive nuclei in a sample to decay and is related to the decay constant by

$$T_{1/2} = \frac{\ln 2}{\lambda}. \quad (1)$$

The classical radioactive decay law assumes that the rate of decay is directly proportional to the number of undecayed nuclei present at any given time. This assumption leads to the first-order linear ordinary differential equation

$$\frac{dN}{dt} = -\lambda N, \quad (2)$$

Where $N(t)$represents the number of radioactive nuclei at time$t$. The analytical solution of Equation (2) yields the well-known exponential decay law

$$N(t) = N_0 e^{-\lambda t}, \quad (3)$$

Where N_0 is the initial number of nuclei? This model has been successfully applied across a wide range of problems due to its simplicity and exact solvability. Despite its broad applicability, the standard radioactive decay equation neglects additional physical processes that may influence decay dynamics in realistic systems. In many nuclear and physical environments, decay may be accompanied by production mechanisms, nonlinear interactions, decay chains, or feedback processes that cannot be adequately represented by a purely linear decay law. Such effects are particularly relevant in complex decay systems, nuclear fuel cycles, population dynamics analogies, and nonlinear physical processes, where deviations from simple exponential behavior may occur.

To overcome these limitations, modified radioactive decay models have been introduced by extending the standard decay equation to include nonlinear terms. One such extension is the modified radioactive decay rate equation given by

$$\frac{dN}{dt} = -\lambda N \pm \beta N^2, \quad (4)$$

Where $\beta$is a parameter that quantifies the strength of the quadratic contribution? The linear term $-\lambda N$ represents the conventional radioactive decay process, while the quadratic term $\pm\beta N^2$ introduces a nonlinear dependence on the number of nuclei. The positive sign corresponds to a production or interaction mechanism that may counteract decay, whereas the negative sign enhances the depletion rate. This formulation allows the model to describe more complex decay–production dynamics, including chain reactions, nonlinear feedback effects, and equilibrium or steady-state solutions, which are not captured by the standard exponential decay model (Cabral and Barros, 2015; Martin and Shaw, 2019).

Analytical solutions of nonlinear differential equations such as Equation (4) are often challenging and may only be obtainable under restrictive assumptions or for special cases. Although closed-form solutions can be derived for some parameter regimes using partial fraction decomposition, the general behavior of the system—particularly for varying parameter values—requires numerical investigation. Consequently, reliable numerical methods are essential for exploring the full dynamics of the modified radioactive decay equation.

Among the available numerical techniques for solving ordinary differential equations, the fourth-order Runge–Kutta (RK4) method is widely regarded as one of the most effective single-step methods due to its high accuracy, numerical stability, and computational efficiency. The RK4 method does not require higher-order derivatives, is self-starting, and provides fourth-order accuracy with relatively moderate computational cost. These properties make it particularly suitable for solving nonlinear decay equations where analytical solutions are either unavailable or insufficient for capturing detailed system behavior.

The application of Runge–Kutta methods to radioactive decay and related physical systems has been well documented in the literature. Ahmed (2001) successfully employed a fourth-order Runge–Kutta algorithm to model fission product accumulation and radioactive decay chains, demonstrating good agreement with established reactor physics codes. Similarly, numerical studies of radioactive decay, population growth models, and nonlinear dynamical systems using higher-order Runge–Kutta schemes have shown excellent accuracy when compared with analytical solutions and improved performance over simpler methods such as Euler's scheme (Anita et al., 2021; Sara et al., 2022; Aroloye and Owa, 2024).

Beyond nuclear physics, equations of the form given in Equation (4) arise in a variety of scientific disciplines. In population dynamics, similar nonlinear terms describe interaction-driven growth or depletion processes. In chemical kinetics, quadratic terms may represent bimolecular reactions, while in applied mathematics and nonlinear wave propagation, such equations model feedback mechanisms and energy transfer processes (Otor et al., 2017; Otor et al., 2018). These cross-disciplinary applications further highlight the importance of understanding the qualitative and quantitative behavior of modified decay equations.

Motivated by the need to accurately model complex radioactive decay processes, this study presents an analytical and numerical investigation of the modified radioactive decay rate equation using the fourth-order Runge–Kutta method. The study aims to examine the influence of the decay constant $\lambda$and the quadratic parameter βon the temporal evolution of radioactive nuclei, to compare numerical solutions with analytical results where applicable, and to identify conditions under which steady-state or equilibrium behavior emerges. By doing so, the work seeks to deepen understanding of nonlinear decay dynamics and to demonstrate the effectiveness of the RK4 method as a robust numerical tool for solving modified radioactive decay models relevant to nuclear physics and related scientific fields.

Theoretical Framework

Radioactive decay is a fundamental nuclear process in which unstable atomic nuclei transform into more stable configurations through the emission of particles or radiation. When large ensembles of nuclei are considered, the decay process can be described deterministically using ordinary differential equations, forming the theoretical basis for analytical and numerical modeling of radioactive systems.

Standard Radioactive Decay Model

The standard radioactive decay law assumes that the probability of decay per unit time is constant and proportional to the number of undecayed nuclei. This leads to the first-order linear differential equation

$$\frac{dN}{dt} = -\lambda N, \quad (5)$$

Where $N(t)$is the number of radioactive nuclei at time $t$and $\lambda$is the decay constant. Solving Equation (5) with the initial condition $N(0) = N_0$ yields the analytical solution

$$N(t) = N_0 e^{-\lambda t}. \quad (6)$$

The decay constant is related to the half-life $T_{1/2}$ of the nuclide by

$$\lambda = \frac{\ln 2}{T_{1/2}}. \quad (7)$$

Equations (5) and (6) describe exponential decay and form the foundation of classical radioactive decay theory.

Modified Radioactive Decay Rate Equation

While the standard decay model is applicable to isolated systems, it does not account for additional mechanisms such as production processes, interaction effects, or nonlinear feedback. To address these limitations, the radioactive decay equation is extended to include a quadratic term, leading to the modified radioactive decay rate equation

$$\frac{dN}{dt} = -\lambda N \pm \beta N^2. \quad (8)$$

Here, β is a parameter that characterizes the strength of the nonlinear contribution. The linear term represents conventional radioactive decay, while the quadratic term introduces nonlinear decay–production dynamics. The negative sign corresponds to enhanced depletion,

$$\frac{dN}{dt} = -\lambda N - \beta N^2, \quad (9)$$

Whereas the positive sign represents a competing production mechanism,

$$\frac{dN}{dt} = -\lambda N + \beta N^2. \quad (10)$$

The inclusion of the quadratic term transforms the governing equation into a nonlinear ordinary differential equation, allowing the model to represent complex decay behavior not captured by the standard exponential law.

Steady-State Condition

For the modified decay equation with a positive quadratic term, equilibrium solutions arise when the rate of decay balances the rate of production. Setting.

$$\frac{dN}{dt} = 0, \quad (11)$$

In Equation (10) gives

$$-\lambda N + \beta N^2 = 0, \quad (12)$$

Which yields the steady-state population

$$N_{ss} = \frac{\lambda}{\beta}. \quad (13)$$

This expression provides the theoretical basis for the steady-state behavior observed in the numerical simulations.

Relevance to Numerical Methods

The nonlinear nature of the modified radioactive decay equations generally precludes simple closed-form solutions for arbitrary parameter values. Consequently, numerical methods are required to investigate the time evolution of the system. The theoretical formulation presented in this section directly motivates the application of the fourth-order Runge–Kutta (RK4) method described in the Methods section, ensuring consistency between the governing equations and the numerical solution procedure.

Each radioactive substance has a characteristic decay period or half-life. A half-life is the interval of time required for one-half of the atomic nuclei of a radioactive sample to decay (Yeşiloğlu, 2019). The radioactive isotope cobalt 60, which is used in radiation cancer therapy, has, for example, a half-life of 5.26 years. Thus, after that interval, a sample originally containing 16 grams of cobalt 60 would contain only 8 grams of cobalt 60 and would emit only half as much radiation. After another interval of 5.26 years, the sample would contain only 4 grams of cobalt 60. Half-lives can range from thousands of years to milliseconds.

Again, it is a common practice to use the *half-life* ($T_{1/2}$) instead of the decay constant (λ) to indicate the degree of instability or the decay rate of a radioactive nuclide. This is defined as the period of time in which half of the radioactivity has disappeared (half of the nuclei have disintegrated) (Li *et al*., 2015).

$$T_{1/2} = \left(\frac{-1}{\lambda}\right) In\left(\frac{1}{\lambda}\right) \quad (14)$$

From which: $\lambda = \frac{In2}{T_{1/2}} = \frac{0{,}693}{T_{1/2}}$

The *mean life* of a nuclide is the sum of the lifetimes of a certain number of nuclei (before they have all disintegrated) divided by the number of nuclei. During the time interval dt, some dN nuclei disintegrate. These "lived" during a period t, which amounts to a total lifetime for dN nuclei (Rozanski *et al.*, 2001) of

$N = N_0 e^{-\lambda t}$

$$t.dN = t.\lambda N.dt \quad (15)$$

Integrating over all nuclei (N) gives the mean life (time):

$$\tau = \frac{1}{N^0}\int_0^\infty t.\lambda N.dt = \lambda\int_0^\infty t.e^{-\lambda t}dt = \lambda\left\{-\frac{t}{\lambda}e^{-\lambda t}\Big|_0^\infty + \frac{1}{\lambda}\int_0^\infty e^{-\lambda t}\,dt\right\} = \lambda\left\{0 + \frac{1}{\lambda}\left(-\frac{1}{\lambda}e^{-\lambda t}\Big|_0^\infty\right)\right\} = \frac{1}{\lambda} \quad (16)$$

As an example, the mean life of a ^{14}C nucleus with $T_{1/2}$ = 5730 a is 8267 years. Then λ = 1/8267, which means that a sample activity decreases by 1‰ in about 8 years; a ^{3}H sample activity ($T_{1/2}$ = 12.43 a) decreases by 5.6% per year.

The modified Inverse Square Model was simulated using the fourth order Runge-Kutta method implemented through the RK4 solver of the Mat lab (Igba and Otor, 2018), and the interaction between lower and upper atmosphere, employing daily data of Total Ozone Column (TOC) decay and atmospheric parameter (cloud cover) over Nigeria from 1998-2012 was investigated by Audu *et al*., 2023 using a decay parameter to evaluate the fluctuation of climate change implemented using RK4. Results reveal that TOC increased spatially from the coastal region to the northeastern region of the country. Pure semiconductors usually possess a single optical absorption within the interband region; the Tauc plot has been a commonly used technique in the estimation of their energy bandgaps (Perverga et al., 2022) following the decay model system particularly RK4 to simulate the movement of electron and holes when they are not in a simple steady state.

To solve this equation numerically, one can use various numerical methods such as Euler's scheme, the Runge-Kutta method, the predictor-corrector scheme, or other advanced numerical techniques like the finite difference method or finite element method. These methods involve discretizing time into small intervals and approximating the derivative, $\frac{dN}{dt}$ using difference approximations, by iteratively updating the quantity N at each time step, one can obtain a numerical solution that approximates the behavior of the modified radioactive decay equation. The specific choice of numerical method will depend on the desired accuracy, computational resources available, and the nature of the problem you are trying to solve.

The equation is likely to be of interest in various scientific and mathematical contexts, such as Radioactive Decay. The equation is a modification of the standard radioactive decay equation, which models the decay of radioactive isotopes (Martin and Shaw, 2019). By introducing the quadratic decay component, it might apply to scenarios involving complex decay processes such as; The population (Dynamics) modeling to describe scenarios where the growth or decline of a population is influenced by both exponential and quadratic factors, in some chemical reactions, the concentration of a species might decay following a similar pattern, and this equation could be relevant in those situations, in applied Mathematics, the equation falls under the domain of ordinary differential equations, and its solutions can be studied and analyzed using various mathematical techniques.

An educational strategy targeting biomedical engineering/physics students focusing on the calculation of isotope concentrations and activities in radioactive decay chains, which is capable of demonstrating the behavior of these isotopes over time, by using an iterative process and basic mathematical operations, was carried out by Ref. (Balbina *et al.*, 2025). The computational modeling of the radioactive decay problem by solving ordinary differential equation systems using the Runge-Kutta fourth-order numerical method was treated.

The specific objectives of the study could include: Analyzing the behavior of the modified radioactive decay equation and understanding the impact of the parameters α and β on the decay process, adapting or developing numerical algorithms to solve the modified radioactive decay equation accurately and efficiently, implementing the numerical methods using programming language or software tool, analyzing and interpreting the results obtained from the numerical solutions, including the decay behavior of the radioactive material under different parameter values and comparing the numerical results with known analytical solution or other reference methods to assess the accuracy of the numerical techniques employed.

The significance of the equation lies in its ability to model more complex decay processes compared to the simple exponential decay equation. Depending on the values of parameters λ and β and the initial conditions, the solutions of this equation might exhibit diverse behaviors, such as stable equilibrium, oscillations, or asymptotic decay, providing valuable insights into the dynamics of the system it represents. The study of this equation can lead to a deeper understanding of natural processes and phenomena in various scientific disciplines. Therefore, we shall analyze the physical or mathematical meaning of the equation and choose appropriate values for λ and β accordingly.

MATERIALS AND METHODS

The modified radioactive decay equation is a fundamental concept in nuclear physics and radiometric dating. It describes the behavior of unstable atomic nuclei as they undergo radioactive decay, which involves the spontaneous transformation of one element into another while emitting radiation. This equation incorporates various factors that influence the rate of decay and is crucial for understanding the age of rocks, minerals, and archaeological artifacts. It can be expressed as:

$$\frac{dN}{dt} = -\lambda N \pm \beta N^2$$

This equation ($where \pm \beta N^2$ $is\ the\ modified\ value$) reflects the probabilistic nature of radioactive decay, with the decay constant determining the likelihood of an individual nucleus decaying in a given unit of time. The higher the decay constant, the faster the nuclei decay, resulting in a shorter half-life. Understanding this equation is essential for various scientific and practical applications, including dating ancient materials, tracking the behavior of isotopes in environmental studies, and ensuring the safety of nuclear reactors and waste storage. It also serves as a reminder of the remarkable and complex nature of the subatomic world, where seemingly unpredictable events can be described and understood through mathematical models. The analytical solution of the standard radioactive decay equation is obtained as follows:

$$\frac{dN}{dt} = -\lambda N \quad (17)$$

$$\int \frac{dN}{N} = -\lambda \int dt$$

$$log_e N = -\lambda t + C \quad (18)$$

If the initial number of nuclei is N_0

And $N = N_0$ when$t = 0$, then (8) becomes

$$log_e N_0 = C \quad (19)$$

$$log_e N = -\lambda t + log_e N_0$$

$$log_e N - log_e N_0 = -\lambda t$$

$$log_e \left(\frac{N}{N_0}\right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$N = N_0 e^{-\lambda t} \quad (20)$$

Also, the analytical solution of the modified radioactive decay equation is obtained as follows:

$$\frac{dN}{dt} = -\lambda N \pm \beta N^2$$

Case I:

$$\frac{dN}{dt} = -\lambda N + \beta N^2 \quad (21)$$

$$\frac{dN}{-\lambda N + \beta N^2} = dt$$

$$\int \frac{dN}{(-\lambda N + \beta N^2)} = \int dt = t + c_1$$

$$\int \frac{dN}{(-\lambda N + \beta N^2)} = t + C_1$$

Integrating the left-hand side using partial fractions

$$\frac{1}{N(-\lambda + \beta N)} = \frac{A}{N} + \frac{B}{(-\lambda - \beta N)}$$

Where, $A = \frac{-1}{\lambda}$ and $= \frac{\beta}{\lambda}$, at $N = N_0$ when $t = 0$ gives

$$N(t) = N_0 = \frac{\lambda}{\beta - e^{\lambda C_1}} \quad (22)$$

Which result to

$$C_1 = \frac{1}{\lambda} In\left(\beta - \frac{\lambda}{N_0}\right) \quad (23)$$

Case II:

$$\frac{dN}{dt} = -\lambda N - \beta N^2 \quad (24)$$

$$\frac{dN}{(-\lambda N - \beta N^2)} = dt$$

$$\int \frac{dN}{(-\lambda N - \beta N^2)} = \int dt + C_2$$

$$\int \frac{dN}{(-\lambda N - \beta N^2)} = t + C_2$$

Using partial fractions to integrate the left-hand side

$$\frac{1}{N(-\lambda - \beta N)} = \frac{A}{N} + \frac{B}{(-\lambda - \beta N)} \quad (25)$$

Where, $A = \frac{-1}{\lambda}$ and $B = \frac{-\beta}{\lambda}$ when $N = N_0$ when $t = 0$, then

$$N(t) = N_0 = \frac{\lambda}{e^{\lambda C_2} - \beta} \quad (26)$$

By rearranging (16)

$$C_2 = \frac{1}{\lambda} In\left(\frac{\lambda}{N_0} + \beta\right) \quad (27)$$

The numerical solution of the modified radioactive decay equation is obtained as follows:

$$\frac{dN}{dt} = -\lambda N \pm \beta N^2$$

Using the Runge-Kutta fourth-order method

$$y_{n+1} = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4] \quad (28)$$

Where,

$$k_1 = hf(x_n, y_n) \quad (29)$$

$$k_2 = \left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right) \quad (30)$$

$$k_3 = \left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right) \quad (31)$$

$$k_4 = \left(x_n + \frac{h}{2}, y_n + k_3\right) \quad (32)$$

Applying the fourth-order Runge-Kutta method to solve the modified radioactive decay equation.

$$\frac{dN}{dt} = -\lambda N \pm \beta N^2$$

$$\frac{dN}{dt} = f(t, N_0) = -\lambda N_0 \pm \beta {N_0}^2 \quad (33)$$

Initializing values; $N_{(0)}$ = the initial value of nuclei

$$N_{n+1} = N_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4] \quad (34)$$

$$k_1 = h.(-\lambda N_0 \pm \beta N_0^2) \quad (35)$$

$$k_2 = h.\left(-\lambda\left(N_0 + \frac{k_1}{2}\right) \pm \beta\left(N_0 + \frac{k_1}{2}\right)^2\right) \quad (36)$$

$$k_3 = h.\left(-\lambda\left(N_0 + \frac{k_2}{2}\right) \pm \beta\left(N_0 + \frac{k_2}{2}\right)^2\right) \quad (37)$$

$$k_4 = h.(-\lambda(N_0 \pm k_3) \pm \beta(N_0 \pm k_3)^2) \quad (38)$$

RESULTS AND DISCUSSION

This section presents the numerical results obtained from solving the standard and modified radioactive decay rate equations using the fourth-order Runge–Kutta (RK4) method. The results are reported strictly in terms of numerical outcomes, parameter values, observed trends, and explicit references to figures. Interpretations, physical explanations, comparisons with previous studies, and broader implications of the findings are intentionally excluded from this section and are addressed in the above section.

Numerical Solution of the Standard Radioactive Decay Model

The standard radioactive decay equation,

$\frac{dN}{dt} = -\lambda N,$

Was solved numerically using the RK4 method for an initial number of nuclei$N_0 = 200$, decay constant$\lambda = 0.1$, and initial time$t_0 = 0$. The numerical solution shows a continuous decrease in the number of radioactive nuclei with time over the simulated interval. Figure 1 presents the numerical solution corresponding to the standard radioactive decay model for$\beta = 0$. Figure 1 should be placed immediately after its first citation here.

Numerical Results for the Modified Decay Equation with a Negative Quadratic Term

The modified radioactive decay equation with a negative quadratic term,

$\frac{dN}{dt} = -\lambda N - \beta N^2,$

Was solved numerically for$N_0 = 200$,$\lambda = 0.1$, and varying values of the quadratic parameter β. Numerical simulations were performed for$\beta = 0.0001$,0.0002 and0.0003. Figures 2–4 show the numerical solutions obtained for these parameter values. In all cases, the numerical solutions indicate a decreasing trend in the number of radioactive nuclei with time. As the value of βincreases, the numerical curves display a steeper decline at earlier times.

Figure 2: $\beta = 0.0001$
Figure 3: $\beta = 0.0002$
Figure 4: $\beta = 0.0003$

Figures 2–4 should be placed immediately after their first citation in this subsection.

Numerical Results for the Modified Decay Equation with a Positive Quadratic Term

The modified radioactive decay equation with a positive quadratic term,

$\frac{dN}{dt} = -\lambda N + \beta N^2,$

Was solved numerically using the RK4 method for$N_0 = 200$,$\lambda = 0.1$ and increasing values ofβ? Simulations were carried out for$\beta = 0.0001$,0.0004,0.00049, and0.0005. Figures 5–8 present the numerical solutions corresponding to these parameter values. The numerical results show a transition in the temporal behavior of the solutions as the value of βincreases. For the highest value ofβ, the numerical solution approaches a constant population level over time.

Figure 5: $\beta = 0.0001$
Figure 6: $\beta = 0.0004$
Figure 7: $\beta = 0.00049$
Figure 8: $\beta = 0.0005$

Figures 5–8 should be placed immediately after their first citation in this subsection and before the beginning of Figure 5.

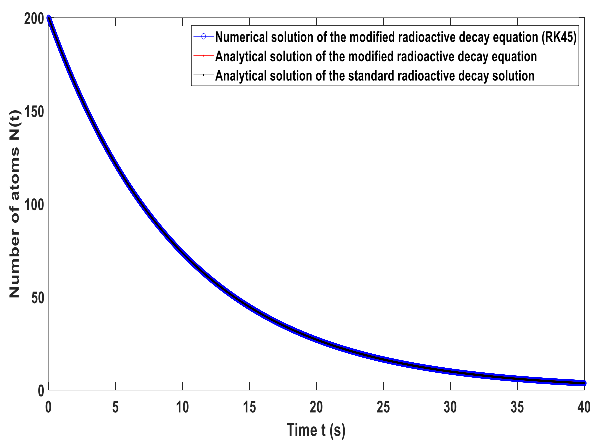

Figure 1: Two Nuclide Decay Equations Solved with the RK4 Method, Where, $N_0 = 200$, $\lambda = 0.1$, $\beta = 0.000$, $t_0 = 0$

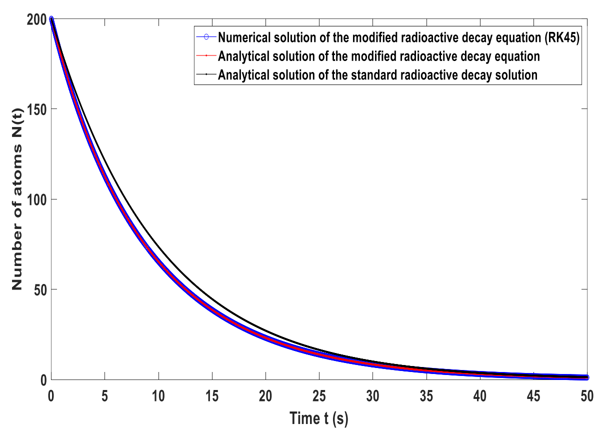

Figure 2. Two Nuclide Decay Equations Solved with the RK4 Method, where $N_0 = 200$, $\lambda = 0.1$, $\beta = 0.0001$, $t_0 = 0$

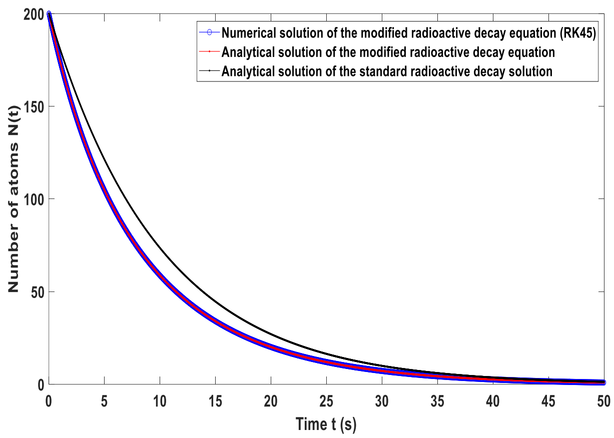

Figure 3: Two Nuclide Decay Equations Solved with the RK4 Method, Where $N_0 = 200$, $\lambda = 0.1$, $\beta = 0.0002, t_0 = 0$

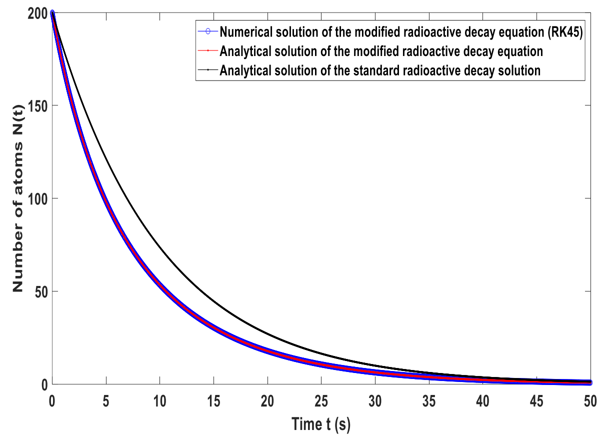

Figure 4: Two Nuclide Decay Equations Solved with the RK4 Method, Where $N_0 = 200$, $\lambda = 0.1, \beta = 0.0003, t_0 = 0$

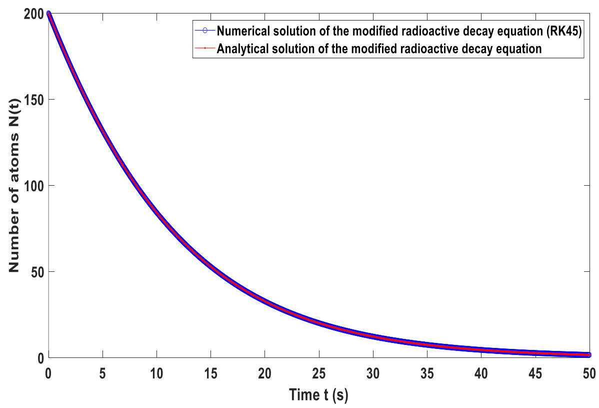

Figure 5: Two Nuclide Decay Equation Solved with the RK4 Method, Where $N_0 = 200$, $\lambda = 0.1$, $\beta = 0.0001, t_0 = 0$

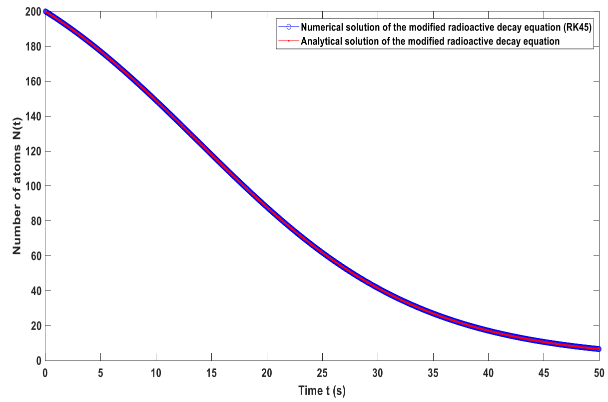

Figure 6: Two Nuclide Decay Equations Solved with the RK4 Method, Where $N_0 = 200$, $\lambda = 0.1$, $\beta = 0.0004, t_0 = 0$

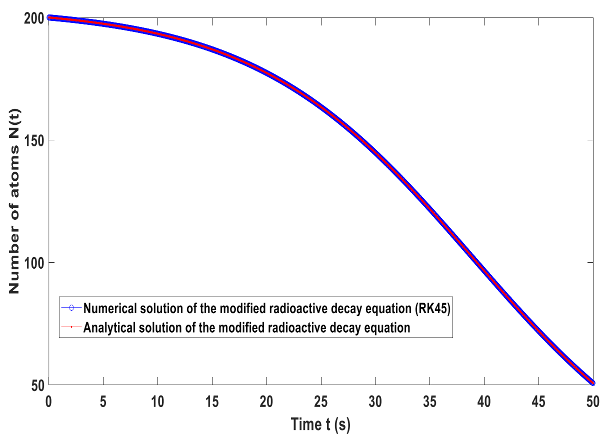

Figure 7: Two Nuclide Decay Equations Solved with the RK4 Method, Where $N_0 = 200, \lambda = 0.1$, $\beta = 0.00049, t_0 = 0$

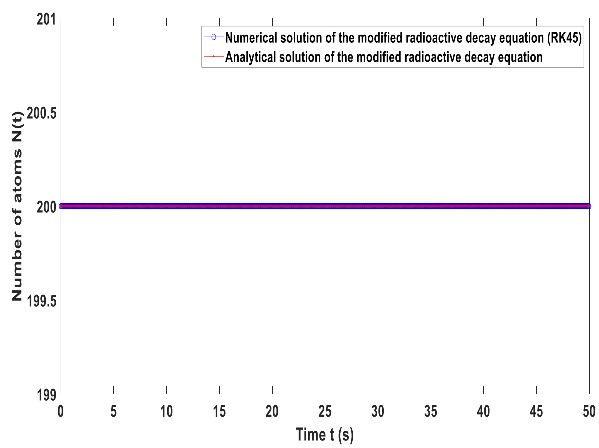

Figure 8: Two Nuclide Decay Equations Solved with the RK4 Method, Where $N_0 = 200$, $\lambda = 0.1$, $\beta = 0.0005, t_0 = 0$

Discussion

The graphs gotten (Figures 1-4) from the solution of this modified radioactive decay equation shows the results of a numerical simulation of radioactive decay using the Runge-Kutta method (RK45). The graphs show the relationship between the numbers of radioactive nuclei remaining as a function of time for two different decay scenarios: simple exponential decay and chain reaction decay. The black curve represents the numerical modified solution (RK45) number of radioactive nuclei remaining as a function of time for the case of simple exponential decay ($\frac{dN}{dt} = -\lambda N$), where the decay rate is constant. The red and blue curve represents the number of radioactive nuclei remaining as a function of time for the case of chain reaction decay ($\frac{dN}{dt} = -\lambda N - \beta N^2$), where the decay rate decreases by a production term that depends on the square of the number of radioactive nuclei.

Simple exponential decay: In simple exponential decay, the rate of decay is proportional to the number of radioactive nuclei remaining. This means that the decay rate decreases as the number of nuclei decreases, leading to an exponential decline in the number of radioactive particles over time. Black curve on the graph illustrates this behavior, showing a continuous decrease in the number of radioactive nuclei over time. Chain reaction decay introduces a new term, $-\beta N^2$, which represents the production of new radioactive nuclei due to interactions between existing nuclei. This term introduces a nonlinearity into the differential equation, leading to more complex decay dynamics. The red and blue curve on the graph captures this interplay between decay and production.

In the modified radioactive equation $\frac{dN}{dt} = -\lambda N + \beta N^2$, the graphs (Figures 5-7) gotten from this radioactive decay equation show the results of a numerical simulation of radioactive decay using the Runge-Kutta method. The graphs show the number of radioactive nuclei remaining as a function of time for chain reaction decay. The red and blue curve represents the number of radioactive nuclei remaining as a function of time for the case of chain reaction decay ($\frac{dN}{dt} = -\lambda N + \beta N^2$), where the decay rate is balanced by a production term that depends on the square of the number of radioactive nuclei.

For a steady state population growth, in the case of chain reaction decay (Figure 8), the red and blue curve plateaus at a steady state value, indicating that the production rate of new radioactive nuclei balances out the decay rate. This steady state population is achieved when the beta value (β) is greater than the decay constant (λ). This is because the production term in the chain reaction decay model balances out the decay term, resulting in a steady-state population of radioactive nuclei. The specific beta value that produces this steady-state population growth was determined by setting the production rate equal to the decay rate and solving for β.

The advantage of using this Runge-Kutta Fourth Order method is that it offers more realistic and simpler solutions and is very easy to computational systems. This condition is similar to what has been mentioned by Balbina *et al.* (2025), compared using adaptive step sizes or Runge-Kutta-Fehlberg (RKF) Methods that use fewer function evaluations. Meanwhile, the weakness of using this method is its high computational cost per step, insufficiency with stiff equations, and the difficulty in implementing adaptive step size control in its basic form. This condition is caused by the decay that occurs in the radionuclide series in the heavy radioactive element series, which requires a very long process to achieve stability.

CONCLUSION

In this research work, two methods for solving a modified radioactive decay equation were carried out: the Standard Radioactive Decay Model, represented by the equation $\frac{dN}{dt} = -\lambda N$ assumes the decay rate is proportional to the remaining radioactive nuclei (N), leading to an exponential decline in their number over time. Modified Radioactive Decay Model represented by the equation $\frac{dN}{dt} = -\lambda N \pm \beta N^2$, introduces a new term (βN^2). This term represents the production of new radioactive nuclei due to interactions between existing ones, adding a nonlinear element to the decay dynamics. However, both models provide valuable insights into radioactive decay, but the modified model offers a more realistic representation in situations with significant new nuclei production. It demonstrates the crucial role of the β value in regulating the steady-state population, highlighting its importance in applications like nuclear power generation and waste management. Based on the literature regarding the numerical solution of radioactive decay equations using the Runge-Kutta fourth-order (RK4) method, the key novel findings and limitations revolve around improved accuracy over traditional methods, computational efficiency, and specific constraints in handling complex physical models. Based on the application of the Runge-Kutta Fourth Order (RK4) method to modified radioactive decay equations, further studies can focus on increasing computational efficiency, improving accuracy, and modeling more complex physical systems.

REFERENCES

Akaagerger, N.B. & Ejembi, S.O. (2025). Effect of Gamma Irradiation on Shelf-Life and Quality of *Musa paradisiaca and Musa acuminata* Locally grown in Benue State Nigeria. *Nigerian Journal of Physics (NJP)*.

34(4): 3027-0936. https://doi.org/10.62292/njp.v34i4.2025.381.

Ahmed, E. A. (2001). Numerical Solution of the Chain Model of Fission Product Nuclides: Effect on the Reactivity of the Reactor. *Annals of Nuclear Energy*. 28(9):23-933. https://doi.org/10.1016/S0306-4549(00)00097-9.

Anita P. P., Trapsilo P., Sri H. B. (2021). Numerical Solution of Radioactive Core Decay Activity Rate of Actanium Series Using Matrix Algebra Method. *Jurnal Penelitian Pendidikan IPA; Journal of Research in Science Education, JPPIPA* 7(3): http://jppipa.unram.ac.id/index.php/jppipa/index.

Aroloye, S.J. & Owa, D.O. (2024). Numerical Solution of Drug Diffusion Model Using Classic Runge Kutta Method. *The Nigerian Association of Mathematical Physics*. 68(2): 117-128. https://nampjournals,org.ng.

Audu O.M., Ejembi E., Otor D.A., Daniel A. O., Chibuoke K.A. (2023). Preliminary Investigation of Total Ozone Column and Its Relationship with Atmospheric Variables. *Atmospheric and Climate Sciences; Scientific Research and Publishing*, 13: 225-239. https://www.script.org/journal/acs.

Balbina, F.C.S, Moraes, F.J.H., Munin, E. & Alves, L. P. (2025). Strategy to Computational Model and Resolve Radioactive Decay Chain in Engineering Education by Using the Runge-Kutta Numerical Method. Access JDreamⅢ for Advanced Search and Analysis. 83(22A1227709):2311-2317. https://jglobal.jst.go.jp/en/detail?JGLOBAL_ID=202202217852831758.

Cabral, V. M., and Barros, L. C. (2015). Fuzzy Differential Equation with Completely Correlated Parameters. *Fuzzy Sets and Systems*, *265*: 86-98. https://www.sciencedirect.com/science/article/abs/pii/S0165011414003741.

Igba, D.S. and Otor, D.A. (2018). Simulation of Earth Planetary Orbits using a Modified Inverse Square Model. *International Journal of Recent Innovations in Academic Research,* 2(1): 23-32, ISSN 2635-3040 (www.ijriar.com).

Li, J., & Jewitt, D. (2015). Disappearance of Comet C/2010 X1 (Elenin): Gone with a Whimper, not a Bang. *The Astronomical Journal*, *149*(4), 133. https://doi.org/10.1088/0004-6256/149/4/133 https://ui.adsabs.harvard.edu/abs/2015AJ....149..133L/.

Martin, B. R., & Shaw, G. (2019). Nuclear and particle physics: an introduction. *Nuclear and ParticlePhysics*. 3(13): 978-1119344612. https://www.amazon.com/Nuclear-Particle-Physics-Brian-Martin/dp/1119344611.

Mumpower, M. R., Surman, R., McLaughlin, G. C., & Aprahamian, A. (2016). The impact of individual nuclear properties on r-process nucleosynthesis. *Progress in Particle and Nuclear Physics*, *86*: 86-126. https://doi.org/10.1016/j.ppnp.2015.09.001.

Otor D. A., Echi I. M. and Amah A. N. (2017). Nonlinear Modulation of Wave Propagation in Spherical Shell Model and Modified Zhang Model using Free Space Model as a Bench-Mark. *Journal ofNatural Sciences Research*, 7(3): 15-28, ISSN :(Paper) 2224-3186 ISSN (Online) 2225-0921. www.iiste.org.

Otor D.A., Echi I.M. and Amah A.N. (2018). Computational Study of Nonlinear Modulation of Wave Propagation in Model Media. *Research Journal of Physical Science*, 6(2): 9-20, ISSN 2320-4796 www.isca.In, www.isca.me.

Perverga, R. J., Obaseki, O.S., Yam, F.K., Stephen, S.M., Avaa, A.A., McAsule, A.A., Yushamdam Y. & Otor, D.A. (2022). Influence of the Secondary Absorption and the Vertical Axis Scale of the Tauc's Plot on optical bandgap Energy. *Journal of Optics (Springer)*, 51(3): 1-10 https://doi.org/10.1007/s12596-022-00961-6.

Rozanski, K., Froehlich, K. and Mook, W.G. (2001). Equations of Radioactive Decay and Growth. In Environmental isotopes in the hydrological cycle: principles and applications, v. III: Surface water, Eds., W.G. Mook, International Atomic Energy Agency Publisher, 3(96), pp 75-77. https://unesdoc.unesco.org/ark:/48223/pf0000124619.

Sara, S.A., Pawan, K. S. & Sunil, K. (2022). A numerical Study of Fractional Population Growth and Nuclear Decay Model [J]. Advances Mathematical Methods for Nonlinear Dynamic System 7(6): 11417-11442. https://10.3934/math.2022637.

Yeşiloğlu, S. N. (2019). Investigation of pre-service chemistry teachers' understanding of radioactive decay: a laboratory modelling activity. *Chemistry Education Research and Practice*, *20*(4), 862-872. https://doi.org/10.1039/C9RP00058E.