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Numerical Computation of Transient Magnetohydrodynamic Micropolar Fluid Flow through a Permeable Surface

^{*1}Hammed, F. A., ²Akanbi, O. O., ¹Usman, M. A., ¹Onitilo, S. A., and ¹Adeyemi, I.

¹Department of Mathematical Sciences, Olabisi Onabanjo University, Ago-Iwoye, Nigeria ²Department of Mathematics and Statistics, Federal Polytechnic, Ilaro, Nigeria

*Corresponding author's email: <u>hammed.fatai@oouagoiwoye.edu.ng</u>

ABSTRACT

The study of magnetohydrodynamic (MHD) micropolar fluid flows through permeable media is gaining increasing relevance due to its broad applicability in industrial and geophysical processes. Micropolar fluids, which exhibit microstructure and microrotation effects, offer a more realistic description of complex fluids such as lubricants, blood, and polymeric suspensions. The present study investigates into the transient phenomenon of a magneto-micropolar fluid configured in a permeable material device. This analysis suits flow in reservoirs, metal casting, and composite manufacturing, petroleum industry, particularly in modelling fluid flow in porous rocks during enhanced oil recovery operations. The understanding of micropolar fluid dynamics in permeable media has been applied to model groundwater flow and contamination remediation strategies. The boundary layer, Boussinesq approximations and some necessary assumptions are used to formulate the mathematical model for the present problem. The model consists of the effects of the non-constant thermophysical properties, viscous and Joule heating properties. The highly coupled nonlinear equations are solved numerically using the unconditionally stable Runge-Kutta Fehlberg method and the shooting techniques. The consequence of the numerical analysis conducted is displayed in various plots and tables for the interpretation of results. The results indicate a reduction in the momentum and thermal boundary structures while the transient term is enhanced is enhanced as reported in the existing literature. The porosity and the magnetic field parameters caused a decelerating motion and raised the thermal distribution as the material term boosts the fluid flow.

Permeable Sheet.

Magnetohydrodynamic,

Non-Constant Viscosity,

Keywords: Micropolar Fluid,

INTRODUCTION

Magnetohydrodynamics (MHD), which studies the dynamics of electrically conducting fluids under the influence of magnetic fields, has gained considerable attention due to its wide range of applications in engineering and biomedical fields. In particular, micropolar fluids, which account for micro-rotation effects and local angular momentum, offer a more realistic description of complex fluids such as blood, lubricants, and liquid crystals. The inclusion of micropolarity enhances the classical Navier–Stokes model by capturing the microstructural behavior of these fluids. When such fluids interact with magnetic fields and permeable or porous surfaces, especially under transient (time-dependent) conditions, their behavior becomes even more intricate and relevant to technological applications such as MHD generators, boundary layer control, cooling systems, and filtration processes. Despite significant progress in steady-state analyses, the transient behavior of MHD micropolar flows over permeable surfaces remains less explored. This study aims to bridge that gap by numerically analyzing the unsteady characteristics of such flows, considering the combined effects of magnetic fields, micro-rotation, and surface permeability.

The non-Newtonian fluids has been found to play crucial roles in various fields of science, engineering and technology ranging from oil drilling, food production, drug manufacturing, paint rheology, medical and biomedical processes. Due to this importance in practical considerations, study have been intensified in the flow

and heat properties of fluids for prediction. The need to enhance industrial productivity and engineering device performance has further motivated various studies on the dynamics of non-Newtonian fluids. Fluid phenomena as reported in (Khan et al., 2019) are a good tool in engineering, biological science, geophysics, and pharmaceutical processes. Fluid flow such as heat transmission over stretching materials, is of utmost relevance in engineering and industrial settings. Such study is of primary importance as the foundation for designing and optimising continuous material-forming processes like polymer extrusion, wire drawing, textile manufacturing, fibre spinning. Improving product quality, process efficiency, and industrial output all depend on an understanding of the nuances of fluid behaviour under these conditions (Makinde et al., 2015; Hsiao, 2016: Khan et al., 2016: Fatunmbi et al., 2020: Fatunmbi and Okoya, 2020).

It is important to remark that various models of non-Newtonian fluids have been developed because no single model has the properties of the fluids in its constitutive equations. Some of these models include micropolar fluid, Casson fluid, Gesekus, Prandtl-Eyring, Powell-Eyring models. Of the models, micropolar fluid stands out due to its ability to simulate complex and complicated fluids of microstructures, rigid particles and nonsymmetric stress tensors. This fluid model was developed by (Eringen, 1966; Eringen, 1972). Its relevance has inspired various scholars to investigate its characteristics. (Das et al., 2016) engaged in Lie group analysis to evaluate the micropolar fluid properties over a vertical plate with slip effects chemical reactions. (Tripathy et al., 2016) simulated the flow of a micropolar fluid in a porous medium. The numerical study incorporated the influence of uneven heat source and concluded that a shrinking structure of the momentum layer exists with a higher magnitude of the porosity and magnetic field terms. (Rashad et al., 2019) discussed such a concept in a porous cylinder consisting of nanoparticles. (Jain and Gupta, 2019) presented out the usefulness of the micropolar fluid in various industrial processes such as liquid crystal solidification, exotic lubricants, colloidal and suspension solution, extrusion of polymer, etc. (Fatunmbi and Okoya, 2021) examined the combined properties of electric-magnetic micropolar fluid flow and heat transfer influenced by dissipative heating and variable physical characteristics in a porous medium. (Gumber et al., 2022) investigated micropolar fluid motion coupled with heat transfer analysis in the presence of thermal radiation, surface mass flux and hybrid nanoparticles. The numerical simulation via the shooting and R-K-F method showed that heat transfer is larger for injection than for suction.

The boundary layer transport and heat transmission in porous devices are of interest in many fields. This is noticeable in, agriculture, where irrigation is practiced, geology, biomedical engineering. To improve the effectiveness and sustainability of these processes, it is essential to have a deeper understanding of the dynamics at play when micropolar fluids interact with inclined, permeable surfaces. Additionally, magnetic fields can be employed to control the motion of fluid and heat distribution and in other engineering works. Such as those found in drug targeting systems for biological systems, the control of ferrofluids in aerospace, treat cancer cells. (Kameel et al., 2014) developed a micropolar fluid flow model over an isotropic porous device using the intrinsic volume averaging idea in light of these applications. (Jabeen et al., 2020) investigated about Newtonian fluid flow through a porous medium membrane subject to viscous and Joule heating using a semi-analytic method. (Fatunmbi et al., 2020) presented the transport of heat-mass of MHD micropolar passing a nonlinearly elongating sheet in a material device filled with pores under the influence of slips. The authors deployed an iterative spectral quasi-linearization method and reported a collapse of the momentum boundary structure with a rise in the nonlinear power index factor. Most of the aforementioned authors have assumed that the flow is steady in their investigations without taking into account the unsteadiness in the flow pattern, which usually occurs in real-life situations. In practical situations, however, unstable flow conditions must be considered because the flow becomes time-dependent due to sudden stretching of the material device or a step change in the temperature or heat flux of the material. In view of those shortcomings, (Elbashbeshy et al., 2012; Yusuf et al., 2021) employed a numerical instrument via Mathematica software to investigate the similarity solutions of an unsteady exponential plate findings. (Nagaraju and Murthy, 2014) explored an unsteady flow of a micropolar fluid in a circular cylinder. (Nadeem et al., 2020) investigated the over-the-flow of micropolar fluid passing an exponentially stretching plate subjected to slip properties. Meanwhile, (Fatunmbi et al., 2022) extended the scenario with entropy generation, several slips, an uneven heat source, and radiation effects. Further, (Khan et al., 2022) presented an unsteady micropolar fluid over a vertical extending and shrinking sheet with a blend of hybridised nanofluid. In their study and those recently presented it was found that the unsteadiness situations in the fluid flow and heat transfer problems capture the intricate behaviour and effective prediction which were ignored in steady-state investigations. This is especially important in practical settings where time-dependent processes or quick changes in external variables play a significant role. The inclusion of porous material opens up a new realm of potential uses in science, engineering, and technology. Thus, the current study have explored a numerical approach to elucidate the complex phenomenon of micropolar fluid flow where the unsteadiness situation is

accounted for. The mathematical model includes the magnetic field term and the interplay between unsteadiness, vortex viscosity dynamics, interactions in porous media, the thermal radiation effect, and weak concentration of the microparticles at the wall. This study is applicable in various scientific and engineering fields, including geophysics and biomedical engineering, to industrial applications such as food processing, extrusion processes, etc.

MATERIALS AND METHODS

Assumptions for Problem Modelling and Governing Equations

In developing the governing equations for the current problem, certain assumptions are put in place. This include, the flow which is assumed to be linear and timedependent. The vivid picture of the configuration of flow and heat dissipation property is depicted in Figure 1. Incompressibility assumption is valid; thermophysical properties change with temperature in a linear fashion, the magnetic field is perpendicular to the motion of the flow, and a weak concentration of the micro-particles is present. Furthermore, the heat region is modelled with the assumption that Joule heating, viscous dissipation and radiative heat sources are present. The Bousnessq assumption is also applied in the momentum equation (2) for the variation of the density in the body force, while the heat distribution at the wall is assumed to be isothermal in nature. Figure 1 shows clearly the sketch of the flow configuration, the coordinate axes of flow and transverse field.



Figure 1: Flow Configuration

In line with those assumptions, the equations governing the present study are;

The symbolic description of various terms in the equations listed are given as follows: *T* depicts the temperature of the fluid, *N* denotes concentration, indicates velocity in the direction of *x*, *v* illustrates velocity in y route, g defines gravitational acceleration, shows the coefficient of thermal expansion, picture coefficient of solutal expansion, κ indicates thermal conductivity, x and y are cartesian coordinates, signifies the microrotation component, n indicates boundary surface term, denotes plate velocity, picture surface flux, is named as specific heat capacity, is known as magnetic field strength, $\gamma = \left(\mu + \frac{\kappa}{2}\right)j$ signifies the spin gradient viscosity, the subscript and connote wall and upstream conditions respectively.

In order to transform partial differential equations to suitable ordinary derivatives, the approach of similarity techniques is employed. Thus, the following similarity quantities are introduced as indicated in equation (7), where is known as the similarity variable, and prime stands for derivatives with respect to but , and are the non-dimensional temperature, stream and concentration in that order.

$$\eta = \sqrt{\frac{c}{v(1-\beta t)}} y, \quad \psi = \sqrt{\frac{c}{(1-\beta t)}} x f(\eta), \quad T = (T_w - T_{\infty})\theta + T_{\infty}, \\ N = (N_w - N_{\infty})\phi, \quad (7)$$

The use of these quantities in (7) in (1-6) results in the validity of (1), whereas (2-6) respectively become $(1 + K)f''' + \alpha\theta'f'' + Kg' + ff'' - Mf' - Daf' +$

$$\begin{aligned} Gr\theta + Gc\phi &= 0, \quad (8) \\ \left(1 + \frac{\kappa}{2}\right)g'' + f'g + fg' - 2K(2g + f'') &= 0 \\ (9) \\ (1 + \delta\theta + Nr)\theta'' + \delta\theta'^2 + (1 + \delta\theta)f\theta' + PrEc(1 + k)f''^2 \\ + PrEcf^2 &= 0 \quad \phi'' + Sc\phi' - Sc\phi'' - (\zeta\phi) &= 0 \end{aligned}$$

Similarly, the conditions at the boundary are now f'(0) = 1, f(0) = fw, g(0) = -nf''(0), $\theta(0) = 1$, $\phi(0) = 1$ $f'(\infty) = 0$, $g(\infty) = 0$, $\theta(\infty) = 0$, $\phi(\infty) = 0$. (12)

The emerging dimensionless parameters are described as follows: denotes the micropolar fluid term, connotes the thermal conductivity term, defines the Prandtl number, defines the Eckert number, defines the Schmidt number, defines the radiation term, stands for the thermal Grashof number, depicts the Solutal Grashof number and depicts the viscosity parameter.



Figure2: Pattern of velocity profile for M

Method of Solution

The governing differential functions of the models yields ordinary differential equations (8-12) after due transformations. These equations designate a boundary value problem and are nonlinear coupled equations. Having noted that, the analytical solution of the equations is not attainable easily, we then resolved to applying a numerical means via the shooting and Runge Kutta Fehlberg method to tackle the problem. This method is unconditionally stable and accurate in tackling nonlinear equations, as reported in (Upreti et al., 2018), (Gumber et al., 2020); and (Das et al., 2016). Thus, computational codes were written for the governing equations and implemented on the Maple software for the solution. The accuracy of the results is checked by verifying the current results with the related studies in literature under strict limiting conditions, as indicated in Table 1. It is found that the results concour excellently well with the published studies of (Ishak et al., 2009) and (Musa et al., 2019) as shown in Table 1, thus, the accuracy of the present result is confirmed.

RESULTS AND DISCUSSION

In this section, the impact of the various emerging terms in relation to the flow fields have been conducted and the results obtained were presented through plots.



Figure 3: Temperature behaviour for M values



Figure 4: Microrotaion profile for M



Figure 6: Thermal reaction to variation in porosity



Figure 8: Thermal field for diverse Pr



Figure 5: Velocity patter for porosity term



Figure 7: Concentration profile for Sc



Figure 9: Velocity profiles for alterations in K values



Figure 10: Velocity profile for unsteadiness term

Figure 2 shows that the magnetic field M resists the fluid motion as it rises in strength. The converse occurs in Fig 3 as the thermal region is enhanced due to higher M. These reactions manifest because of the Lorentz force imposed on the fluid. It can be seen that magnetic field inclusion in the flow of the micropolar fluid can effectively control the flow pattern and the heat distribution as well. The microrotation field is found to be increasing with the action of M. The boundary layer structure of the microrotation profile continues to enlarge when micro-particles are present, as shown in Figure 4. Figure 5 illustrates the effect of the porosity parameter on the velocity profile. The result indicates that increasing the porosity parameter tightens the porous medium and consequently increases the resistance against the flow, thereby decreasing the fluid velocity profile. Figure 6 depicts the temperature profile for various values of the porosity parameter. It is noticed that the temperature profile increases with an increasing porosity parameter. This is due to the fact that as fluid flow slows down resulting from the tightening of the porosity medium, heat is transferred to the fluid from the hot surface and thereby increasing the temperature profile. It is evident in Figure 7 that rising values of Sc result in a depletion in the solute profile. The behaviour of Pr in the thermal vicinity is presented in Figure 8. A depleted structure of the thermal boundary surfaced as Pr enlarges, and such a



Figure 11: Velocity profile for Grashof

reaction resulted in a diminished temperature distribution in the heat region, as clearly noted in the plot.

A rise in the velocity profiles and the hydrodynamic structure is found to occur in Figure 9 as K grows in magnitude. This is due to the reduction in the dynamic viscosity which lessens the resistance on the fluid motion The effect of the unsteadiness parameter on the velocity profile is presented in Figure 10. It is observed that increasing the unsteadiness parameter leads to a decrease in the velocity profile. This is due to the thickening of the thermal boundary layer as the unsteadiness parameter increases, which in turn decreases the velocity profile. On the other hand, there is an accelerating flow as rises. This occurrence is observed due to the buoyancy force which is encouraged by the growth in while the viscous force is diminished.

Results verification

In this section, the validity and accuracy of the result obtained from the present study is conducted. The numerical results for Nusselt number with existing studies is also observed for exceptional cases. The results have been verified by comparing the values of the Nusselt number with the existing results in the literature under some limiting constraints. As indicated in Table 1, the present results is found t be in good agreement with the existing results.

Α	fw	Pr	Gr	Ishak et al.	Musa	Present study
				(2009)	et al.	
					(2019)	
0	-1.5	0.72	0	0.4570	0.4570	0.457297
		1.00		0.5000	0.5001	0.500017
		10		0.6452	0.6451	0.645160
	0	0.01		0.0197	0.0197	0.112200
		0.72		0.8086	0.8086	0.808834
		1.00		1.0000	1.0000	1.000008
		3.00		1.9237	1.9239	1.923679
		10.0		3.7207	3.7207	3.720671

Table 1: Comparison of $\theta'(0)$ for variation in *A*, *fw*, Pr and Gr

CONCLUSION

The present problem is based on the unsteady motion of magnetized micropolar fluid over a vertical stretching material in a porous device. The flow is configured in a two-dimensional sheet with isothermal heating conditions and chemical reaction terms. The other parameters included in the study are thermal radiation changeable thermophysical properties, A mathematical model is set up for the design of the problem, and the equations are subjected to a numerical solution via the Runge-Kutta Fehlberg method combined with the shooting techniques. The investigation outcomes are shown in Table 1 and plots to determine the intricate relationship of the physical terms on the transport phenomenon. The investigation reveals that the porous parameter, unsteadiness, and magnetic field terms deplete the flow of the fluid. There is a significant reduction in the surface drag tension as the material and magnetic field terms rise in magnitude. Heat transfer is improved by the micropolar parameter but reduced by the magnetic field term.

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