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Theoretical Equation of State for Neutron Stars, its Maximal Mass in the Framework of General Relativity before Collapsing into a Black Hole

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ABSTRACT
Despite significant advancements in multi-messenger astronomy and gravitational wave detection, the accurate determination of EOS for NS remains a critical challenge in astrophysics. Current theoretical models often fail to comprehensively reconcile the fundamental properties of NS such as the maximum mass, with observational evidence. The gap limits the full understanding of the maximum stable mass and density of NS, as well as it behavior under extreme relativistic conditions. Computational tools like Einstein's toolkit and recent multi-massagers observations by LIGO and Virgo have provided useful data, there is a lack of unified, precise EOS models' that incorporate both theoretical and observational constraints. Addressing this problem is essential for advancing our understanding of NS physics and for guiding future observations. This research therefore, aims to address these gaps by constructing a robust theoretical EOS models for NS using piecewise polytrope approach based on general relativity, supported by computational simulations and validate same against existing observational data. An EOS constructed from theoretical models and numerical simulations has revealed that a NS can attain a maximum mass of $2.33M_{\odot}$ before collapsing into a BH at a radius of up to 12 km, based on the mass-radius relationship derived from the model.

Keywords:
Equation of state,
Neutron star,
Mass,
LIGO,
Virgo.

INTRODUCTION

One of the most interesting topics in research is the investigation of severe environments around the globe. A significant milestone in contemporary astronomy was reached in 2017 when the Laser Interferometer Gravitational-wave Observatory (LIGO) detected gravitational waves for the first time from the merging of two neutron stars (NSs) (LIGO Scientific Collaboration, 2021).

It confirmed Albert Einstein's century-old prediction in 1915 (Einstein, 1915) that masses moving through space would create ripples in its fabric, which travel at the speed of light. Neutron stars (NSs) act as cosmic laboratories where new forms of matter could exist, including hyperons and quarks. Through experimental methods and theoretical models, scientists have expanded our understanding of hot and dense matter by studying exotic nuclei, pushing the boundaries of knowledge in nuclear experiments and astrophysical observations (Abbot et al., 2021). Abrupt supernova explosions of dying giant stars produced the immensely dense objects known as NSs. Most importantly for NS theory, it is the equation of state (EOS) of dense matter in the interiors of NS, Pressure (P) and mass density (ρ) or related energy density are related and this relationship is known as the EOS.

The dependence of P (ρ) is necessary for constructing NS models and is considered a fundamental parameter in this study (Agathos et al., 2015). Despite extensive research, the EOS of neutron stars is still not precisely known due to their extreme properties, which make them ideal laboratories for potentially discovering new phases of matter. These exotic systems have been utilized to test various predictions of the theory of General Relativity (GR) with unprecedented accuracy, with direct detection of gravitational waves (GW) emitted by objects such as NSs and black holes (BH). GW are ripples in space time created by the motion of massive objects like NSs (Radice et al, 2018; Abbott et al., 2021).

Despite significant advancements in multi-messenger astronomy and gravitational wave detection, the accurate determination of EOS for NS remains a critical challenge in astrophysics. Current theoretical models often fail to comprehensively reconcile the fundamental properties of NS (e.g. mass, radius, pressure, and density) with observational evidence. The gap limits the full understanding of the maximum stable mass and density of NS, as well as its behavior under extreme relativistic conditions.

This study however, focuses on the object, to construct a theoretical equation of state for neutron star using general relativity approach and piece-wise polytrope model to describe their maximum mass before it becomes unstable and collapses into a black hole.

Theoretical Framework

According to the geometrical framework of a four-dimensional manifold, General Relativity (GR) is established ((Einstein, 1915); the four coordinates ($\chi_{0,}\chi_{1,}\chi_{2,,}\chi_{3,}$) are used to identify an event in this manifold. $\chi_{0,}$ which corresponds to time t, ($\chi_0 = \mathrm{ct}$) with c being speed of light, and the remaining coordinates, ($\chi^{i} = 1,2,3,$) indicate the spatial location. One of the main outcomes of GR is the Einstein field equations (Shapiro & Teukolsky, 1983). Through affine connections, they establish a connection between the gravitational potential and the related metric g and the physical properties of matter, which are represented by the energy-momentum tensor T. These differential equations decreased to the Poisson equation at the Newtonian limit given by.

$$\nabla^2\phi = 4\pi G\mu_o \qquad (1)$$

where the mass density is represented by μ_0, the universal gravitational constant is G, and the Newtonian gravitational constant is ϕ (Shapiro & Teukolsky, 1983). This limiting situation indicates that the second derivatives of the gravitational potential are contained in the Einstein tensor $G_{\mu\nu}$. In fact

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}, \qquad (2)$$

in which $R = g^{\mu\nu}R_{\mu\nu}$ the scalar curvature is represented as $R_{\mu\nu}$, while the Ricci tensor is identified as $R_{\mu\nu} = R^{\sigma}_{\mu\sigma\nu}$, which is a contraction of the Riemann tensor. On a closed curve, the Riemann tensor indicates the influence of curved space in parallel conveying vectors. It can be represented as a function of Christoffel symbols $\Gamma^{\sigma}_{\nu\mu}$, and is calculated as a commutator between covariant derivatives in two space-time directions.

$$\mathrm{R}^{\sigma}_{\nu\mu} = \partial_\tau\Gamma^{\sigma}_{\nu\mu} - \partial_\nu\Gamma^{\sigma}_{\tau\mu} + \Gamma^{\sigma}_{\tau\gamma}\Gamma^{\gamma}_{\nu\mu} - \Gamma^{\sigma}_{\nu\gamma}\Gamma^{\gamma}_{\tau\mu} \qquad (3)$$

where $\Gamma^{\sigma}_{\nu\mu}$ is given as

$$\Gamma^{\sigma}_{\nu\mu} = \frac{1}{2}g^{\sigma\gamma}(\partial_\mu g_{\nu\gamma} + \partial_\nu g_{\mu\gamma} - \partial_\gamma g_{\nu\mu}) \qquad (4)$$

Here the notation ∂_σ to label $\frac{\partial}{\partial x^\sigma}$ $with\ \sigma = 0,1,2,3$. The field equations from Einstein are:

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \quad \mu,\nu = 0,1,2,3 \qquad (5)$$

Finding the solutions to the current theories of gravity and comparing them to physical objects like BHs and NSs, which are solutions of these theories consisting of space-time disturbances propagating at the speed of light c, is one of the core goals of these theories. The nuclear EOS can be inferred from the GW emission of a binary neutron star (BNS) inspiral. The tidal polarizability characteristics of the two NSs provide this information, which is most noticeable in the late inspiral period right before the merger. The amplitude and frequency of the GW inspiral signal rise when BNSs evolve as a result of the gravitational radiation reaction, reaching a merger frequency of about $2000\,Hz$. The goal of GW signal observation from BNS systems is to improve our comprehension of the unknown EOS of NS matter and the structure of NS. This work investigates the possibility that the inspiral signal parameters provide useful data regarding the EOS of materials from neutron stars (Haensel, et al, 2006; Camenzind, 2007).

Piecewise Polytrope Model: When it comes to dense matter models, the main issues that scientists are dealing with are first the difficulty of obser ving the matter and second, the difficulty of accurately c omputing many-body interactions. Constraining the EOS of ultra-dense materials can be done in two ways. Phenomenological models that are modified to fit observations of NSs and nuclear events are based on density-dependent interactions.

Recall the hydrostatic equilibrium second-order deferential equation, one need to determine the relation between pressure and density;

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho_{(r)}}\frac{dp}{dr}\right) = -4\pi G\rho_{(r)} \qquad (6)$$

and that is where we are going to use what is called polytrope solution to solve which is expressed in equation (7)

$$P = K\rho^{\Gamma} \qquad (7)$$

In the model expression, the pressure as being equal to a constant K times the density raised to another constant gamma Γ and $\Gamma \equiv 1 + \frac{1}{n}$, n, is polytrope index, and it can take any number. take note that Γ is defined by the polytrope index and Γ_1 is an adiabatic index.

Let's defines the radial coordinate,

$$r \equiv \alpha.\xi.\frac{d}{dr} = \frac{d}{d_{(\alpha\xi)}}, \qquad (8)$$

ξ is a dimensionless variable a proxy for radius and α is a new constan

In the polytrope model, the density; $\rho = \rho_o\theta^n$, $\theta = \theta(\xi)$ then plug it into (7)

$$\rightarrow \quad P_{(\xi)} = K\rho_o^{1+1/n}.\theta_{(\xi)}^{n+1} \qquad (9)$$

Concerning the matter's compressibility, three distinct groups can be applied when organizing the corresponding EOSs: soft, mild, and stiff. According to research by Vivanco, et al. (2019), with varying EOSs, one can generate a variety of stellar models, especially concerning maximum masses, which range from M_{max}~$1.4M_{\odot}$ for the softest EOSs to M_{max}~$2.5M_{\odot}$ for the stiffest. The EOSs can also be separated based on the matter's composition; only nucleon matter can be responsible for extremely stiff EOSs. (Friedman, et al, 1984).

Star models can be computed in the framework of General Relativity after obtaining the EOS. The central density ρc is used to parameterize a family of neutron star models, where the gravitational mass $M = M(\rho c)$ and the circumferential radius $R = R(\rho c)$ are derived. The proper length of the neutron star equator is represented by $2\pi R$. (Fujimoto, et al, 2021).

$$\frac{dp}{dr} = -\frac{Gm_{(r)}}{r^2}.\left(1+\epsilon+\frac{P}{\rho_o c^2}\right)\left(1+\frac{4\pi r^3 P}{m_{(r)}c^2}\right)\left(1-\frac{2Gm_{(r)}}{rc^2}\right)^{-1} \quad (10)$$

From the derived TOV equation (10), where p = pressure, ρ_o = rest mass density, $m_{(r)}$ = enclosed mass, and $\left(1-\frac{2Gm_{(r)}}{rc^2}\right)^{-1}$ is the metric deviation; For a NS we assume zero temperature limit and a constant density, now NS actually have a high temperature, so the zero-temperature limit does not actually mean the temperature is zero, it means that it is low compared to the chemical potential. Constant density can be interpreted in two ways, either we take equation (11) constant or equation (12) constant.

$$\rightarrow \ \rho_o, (1+\epsilon) = \text{constant} \quad (11)$$

$$\rightarrow \ \rho_o = \text{constant and } \epsilon = 0 \quad (12)$$

Defining:

$$x = \frac{P}{\rho c^2}, \quad \beta_{(r)} \equiv \frac{2Gm_{(r)}}{rc^2} \quad (13)$$

Here β is metric deviation, plug this (13) into equation (10) gives

$$\rightarrow \ \frac{d(\rho c^2 x)}{dr} = -\frac{\rho}{2r}.(1+x)(\beta c^2 + 8\pi G r^2 \rho x)(1-\beta)^{-1} \quad (14)$$

Since the density is constant, we have mass is just the volume times the density ρ. We can plug this into β to find that β is proportional r^2.

$$m_{(r)} = \frac{4\pi r^3}{3}\rho \quad (15)$$

and

$$\beta_{(r)} = \frac{8\pi G\rho r^2}{3c^2} \quad (16)$$

Note, $8\pi G r^2 \rho = 3\beta c^2$

Then,

$$\rightarrow \ \beta_{(r)} = \frac{8\pi G\rho r^2}{3c^2} \rightarrow \frac{d\beta}{dr} = \frac{2\beta}{r} \quad (17)$$

Using the chain rule,

$$\rightarrow \ \frac{d}{dr} = \frac{d\beta}{dr}, \frac{d}{d\beta} = \frac{2\beta}{r}.\frac{d}{d\beta} \quad (18)$$

Then plug (18) into equation (17)

$$\Rightarrow \ \rho c^2.\frac{2\beta}{r}\frac{dx}{d\beta} = -\frac{\rho}{2r}.(1+x)(\beta c^2 + 3\beta c^2 x)(1-\beta)^{-1} \quad (19)$$

Some term cancelling out and rearranging,

$$\Rightarrow \ \frac{dx}{(1+x)(1+3x)} = -\frac{1}{4}.\frac{d\beta}{1-\beta} \quad (20)$$

Then integrate equation (20) gives

$$\rightarrow \ ln\left(\frac{1+3x}{1+x}\right) = \frac{1}{2}.\ln(1-\beta) + C \quad (21)$$

Find the constant, assume the $\rho_s = 0$, means that $x(r = R) = 0$.

$$\Rightarrow \ ln\left(\frac{1+3x}{1+x}\right) = \frac{1}{2}.\ln(1-\beta) + C \quad (22)$$

$\rho_s = 0 \ \rightarrow x(r = R) = 0$

Then we define another quantity, $\bar{\beta} \equiv \frac{2Gm}{Rc^2} \rightarrow x(\bar{\beta}) = 0$

Then plug in $c = -\frac{1}{2}\ln(1-\bar{\beta})$ in equation (21) and re write

$$\Rightarrow \ ln\left(\frac{1+3x}{1+x}\right) = ln\sqrt{\frac{1-\beta}{1-\bar{\beta}}} \quad (23)$$

Solving for $x(\beta) \rightarrow P(\beta) = \rho c^2.x(\beta)$

$$\Longrightarrow \quad P(\beta) = \rho c^2.\frac{\sqrt{1-\beta}-\sqrt{1-\bar{\beta}}}{3\sqrt{1-\bar{\beta}}-\sqrt{1-\beta}} \quad (24)$$

$\beta = \frac{2Gm_{(r)}}{rc^2} = \frac{8\pi G\rho r^2}{3c^2}, \quad \bar{\beta} = \frac{2Gm}{Rc^2}$

and central pressure:

$$\rho_o \equiv p_{(o)} = \rho c^2.\frac{1-\sqrt{1-\bar{\beta}}}{3\sqrt{1-\bar{\beta}}-1} \quad (25)$$

Note the denominator if it $\rightarrow 0$; $p_{(o)} \rightarrow \alpha$ *and if* $\bar{\beta} \rightarrow \frac{8}{9}$, the star will collapse to black hole.

The maximum mass of NS can be, when $\frac{2Gm}{Rc^2} = \frac{8}{9}$, rewriting the radius gives;

$$R = \left(\frac{3M}{4\pi\rho}\right)^{1/3} \quad (26)$$

assume the nuclear density:

$$\rho \approx 2.\frac{2M_n}{4\pi r_n^3} \quad (27)$$

here r_n is nuclear radius $\approx 10^{-15}m$. Neutron has two spin state, spin up/down, hence the 2. substituting the density (27) into the radius (26)

$$\Longrightarrow \ \frac{2Gm}{Rc^2} = \frac{2G}{r_n c^2} \ .(m^2 \cdot M_n)^{\frac{1}{3}} = \frac{8}{9} \quad (28)$$

Now solve for the mass thus as;

$$M_{max} \approx \frac{1}{\sqrt{2M_n}}.\left(\frac{r_n c^2.8/9}{2G}\right)^{\frac{3}{2}} \gg 3M_{\odot} \quad (29)$$

MATERIALS AND METHODS

Model Setup: The intent of this study is in solving the TOV equation, which in the framework of GR describes the structure of a static, spherically symmetric NS (Oppenheimer, & Volkoff, 1939). Combining theoretically derived parameters from the nuclear matter EOS for NS with observationally obtained parameters Table 1 was developed. A customized Python package was developed and adapted from previous codes to enable the study of static neutron stars with various EOS and to calculate tidal properties. This package was specifically designed for ease of use in a Jupyter

Notebook environment, where it can be modified, executed, and visualized interactively and the schematic summary of the work flow is shown in figure 1. The package implements key modifications, allowing it to:
Solve the TOV equation: It computes the internal structure of NS based on the input EOS, mass, density, pressure, and radius.
Perform interactive visualizations: The package outputs detailed graphs for the Mass-Radius, Pressure-Density, Mass-Density, and Radius-Density relationships, enhancing the analysis of EOS models.
The simulations and code modifications were carried out on an Ubuntu 24.12.04 system using kuibit a Python 3.12 library and the Jupyter Notebook environment for script writing, debugging, and visualization. The kuibit library was utilized to analyse and visualize the output from the Einstein Toolkit (Löffler, et al., 2012) simulations, while the Wolfram Mathematica was employed for additional computations where high accuracy was required.

RESULTS AND DISCUSSION

Results and Comparison with other researchers EOS

The M(R) connection is a crucial parameter in NS observations. To determine the EOS of NS matter at very high densities, it is essential to know the maximum gravitational mass of an NS, which indicates its stability against collapse into a black hole. Based on our simulations, figure 1 illustrates the maximum NS mass as a function of radius.
The detection of GW from the merger of two neutron stars in 2017, designated GW170817, marked the beginning of a new era in multimessenger and multiwavelength astronomy. This event, provided astronomers with a powerful tool to refine our understanding of NS physics. The constraints imposed by GW170817, combined with galactic measurements, have led to a revised mass limit of $2.0M_{\odot}$; this finding effectively rules out any EOS for NSs that exceeds $2.0M_{\odot}$. Variations in the radius and mass can impact how NSs emit radiation, interact in binary systems, or produce gravitational waves.

Table 1: The Nuclear matter EOS parameters of NS

S/N	Mass ($M_{\odot}$)	Radius (Km)	Densities (gcm^{-3}	Γ	K
1	1.44	14	4×10^{4}	3.00	100
2	2.00	13	2×10^{11}	2.50	100
3	2.50	12	3×10^{14}	2.00	100
4	3.00	11	2×10^{16}	1.25	100
5	3.50	9	5×10^{16}	1.00	100

The Maximal Mass

An evolutionary framework for the development of NSs was aided by the initial suggestion by Bethe and Johnson (1974) that NSs form towards the conclusion of the life of large progenitor stars and by their known correlations with supernova remnants. NSs are thought to be generated by the collapse of an iron (56Fe and neighbouring isotopes) core that originated at the centre of progenitor stars with initial masses ranging from $8\ to\ 25M_{\odot}$, despite an initial diversity of theories being put forth.
Even with early mass measurements, an inaccurate association between the iron core before the collapse and the Chandrasekhar limit led to the establishment of a paradigm for a distinct formation pathway for NSs, with a mass scale of about $1.4M_{\odot}$ and low dispersion.

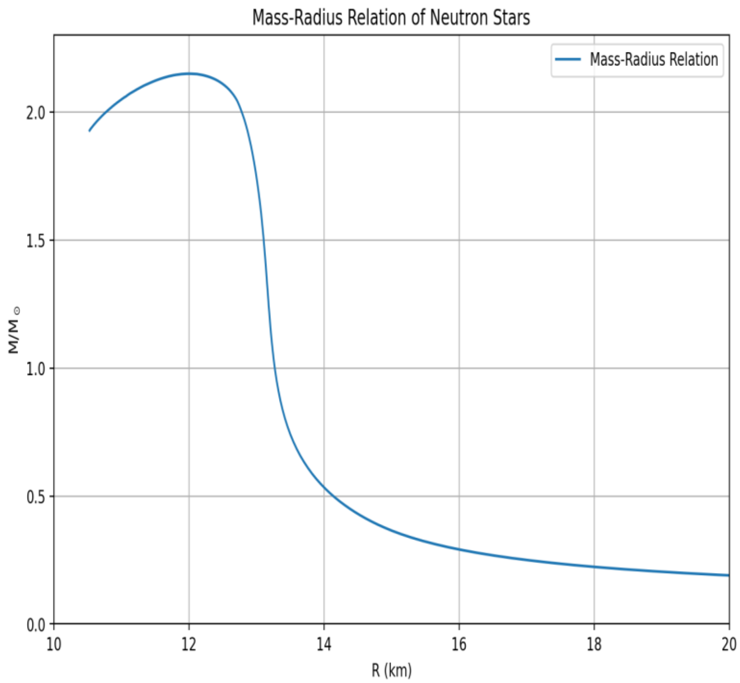

Figure 1: Mass-Radius Relation

The mass-radius relationship is illustrated in Figure 1, showing how the mass of an NS changes with its radius. Initially, the radius increases with mass, but it eventually decreases due to gravitational collapse. According to the simulation result, the maximum mass indicated in the figure is $2.33M_{\odot}$ at a radius of 12 km, which corresponds to a soft EOS. This relationship is non-linear due to the effects of the EOS. The findings suggest that neutron stars can exceed a mass of $2.0M_{\odot}$ before collapsing into a BH, a value that was ruled out by observations from the NS merger GW170817 since the value obtained is supported by current astrophysical models.

These results highlight the balance between gravitational forces and the pressure generated by the dense matter within the NS. The curve in Figure 1 represents the maximum mass that an NS can sustain before succumbing to gravitational collapse, a process that can lead to the formation of black holes.

Assuming the equation of state above a certain density to be as stiff as possible, Rocha, et al (2023) established an upper threshold for NS masses at $3.2M_{\odot}$, which has since been adopted to distinguish NSs from BHs. Although there was room for high masses due to uncertainties in the early X-ray mass measurements, theoretical studies suggested that a few physical EOSs could lead to a maximum mass of about $2M_{\odot}$, the scientific community eventually came to the consensus that NS masses should not exceed the "canonical" value of $1.4M_{\odot}$, which is consistent with the first accurate mass measurements, for evolutionary reasons. Alternatively put, the value imprinted at birth by collapse physics was $1.4M_{\odot}$ (Baym, et al, 1971).

Nonetheless, over time, observational efforts have resulted in a steady increase in the number of recorded masses. The mass range that NSs encompass has been known for more than ten years; the present interval extends from $1.17M_{\odot}$to values greater than $2.0M_{\odot}$, a far wider range than was previously believed to be conceivable.

Comparison with other researchers EOS

The results of this research were compared to various Equations of State (EOS) from other researchers in the literature who employed different numerical models. These models varied in their underlying assumptions, computational methods, and parameter choices. The EOS used by these researchers were tested under similar conditions, though the precise temperature, pressure, and material properties may differ. Table 2 presents the corresponding plots of each EOS, while Figure 2 illustrates these results graphically, highlighting key trends across the various models.

The simulation results of this study show close agreement with those of one researcher, whose model employed a similar approach to handling thermodynamic interactions and physical properties, leading to comparable predictions. However, the findings from five other researchers differ significantly, as shown in Table 2. These discrepancies may arise due to variations in the numerical methods (e.g., finite element vs. finite difference methods), differences in boundary conditions, or the use of different reference data. In particular, variations in assumptions about the ideality of the gas or the form of the intermolecular potential may have contributed to the observed differences in the results.

Table 2: EOS of other researchers

S/N	EOS	Γ	$\rho(gcm^{-3})$
1	SLy	2.488	1.462×10^{14}
2	AP3	3.330	5×10^{17}
3	ALF4	2.400	0.888×10^{14}
4	BBB3	2.909	0.942×10^{14}
5	WFF3	3.224	$10^{14.7}$
6	GNH3	2.610	10^{15}

The interior of an NS consists of two main sections: the crust and the core. The crust is nearly as dense as nuclear matter, indicating it is primarily composed of nucleons. However, the core's composition and interactions remain poorly understood, despite extensive research, as it is significantly denser than nuclear density. It's expected that the emergence of exotic particles will trigger various phase changes in the core's high-density matter. Many studies have compared their findings with others on the EOS of an NS matter, utilizing different methods to derive the EOS at specific densities, radii, and masses.

Understanding a neutron star's structure and the physics driving its interior requires knowledge of its maximum mass and radius. Since it controls how matter behaves in extreme circumstances, the EOS has a direct impact on these parameters. Neutron star observations, especially those using pulsar timing and gravity waves, have yielded information suggesting upper limits on mass, usually in the range of $2 - 2.5M_{\odot}$. These results imply that the star's radius reduces as the mass gets closer to this limit, which could eventually, result in possible phase transitions in the core. Investigating these correlations advances our understanding of neutron stars and clarifies basic interactions in dense matter.

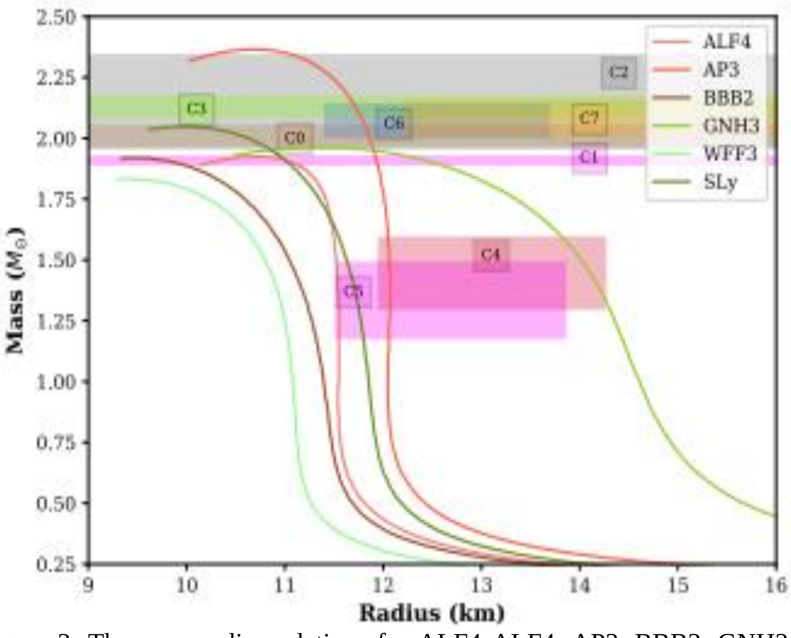

Figure 2: The mass-radius relations for ALF4 ALF4, AP3, BBB2, GNH3, WFF3 and SLy EOSs. The shaded regions depict the imposed mass-radius constraints from astronomical observations (Singha, Vaneshwar, & Kumar, 2022)

CONCLUSION

A theoretical and numerical simulation-based EOS has shown that a NS can reach a maximum mass of $2.33M_{\odot}$ solar masses with a radius of up to 12 km, beyond which it is predicted to collapse into a black hole (BH). This finding, represented by a mass-radius curve, reflects the balance between gravitational pressure and internal forces that resist collapse. The model accounts for various factors, such as the nuclear equation of state, exotic matter at high densities, and relativistic corrections, which influence the star's structure and stability. Understanding the mass-radius relationship is crucial for determining the maximum stability limits of NSs, providing key insights into the formation and evolution of compact objects and aiding in the detection of gravitational waves from NS mergers. Future research should focus on refining EOS models to better capture the behavior of neutron star materials under extreme conditions, such as high densities where exotic particles like quarks and hyperons play a role. Such advancements would offer more precise predictions of neutron star characteristics, including mass, radius, and stability limits. Additionally, exploring the effects of extreme temperatures or strong magnetic fields on the structure and stability of NSs could further enhance our understanding of their formation, particularly in the context of supernovae or neutron star mergers.

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