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Impact of Relaxation Time from Improved Bloch NMR Fluid Flow Equation on NMR Signal of Blood Proton Spins using Laplace Transform Method

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ABSTRACT
Nuclear Magnetic Resonance (NMR) is a non-destructive spectroscopic technique in which the magnetization of a sample, such as human blood, changes by the application of both an external and radio frequency (RF) field to the sample. NMR is a non-invasive tool used in medicine and surgery for getting valuable information on blood flowing along the human blood vessels. In the existing work done on NMR signal, which is the transient state solution of the Bloch NMR fluid flow equation, it is noted that some NMR blood flow parameters, such as static magnetic field and Larmor precessional frequency of blood-spinning protons, are missing in the existing NMR signal equation. In order to solve this problem to generate accurate NMR signal results. This study developed a new NMR signal equation that contains the missing NMR parameter. Furthermore, the signal equation was used to investigate the effect of transverse relaxation times of arterial, venous, and capillary blood, respectively, on the NMR signal of blood proton spins. Laplace transform method was utilized to obtain the NMR signal, which is the closed-form solution of the improved Bloch NMR fluid flow time-dependent equation. MATLAB and Origin Pro software tools were utilized for the generation and simulation of data. Results of the simulation and analysis showed that for varied relaxation times of arterial, venous, and capillary blood, the NMR signal generated from the blood proton spins decreased with increasing time. This means that the blood proton spins gradually lose their alignment of magnetic moments, which is due to the interactions between neighboring blood proton spins. The results obtained from the simulations may be used as a means of validating the experimental results obtained from an NMR/Magnetic resonance imaging machine by providing a theoretical basis for comparison with the experimental data.

Keywords:
Relaxation time,
Bloch NMR fluid flow equation,
Blood vessels,
Laplace transform,
NMR signal.

INTRODUCTION

Nuclear magnetic resonance (NMR) is a powerful and versatile technique in which magnetic properties of the atomic nuclei of a material, for instance human blood, are excited by the application of both an external magnetic field and a radio frequency (RF) magnetic field. This excitation makes the blood nuclei to precess or spin around the axis of the applied magnetic field, producing a detectable signal that provides important information about the chemical properties of the human blood spinning nuclei (Khan *et al.*, 2022; Rasheed & Usman, 2023).

NMR can be mathematically defined and explained by using the Bloch NMR fluid flow equation. This is a linear differential equation that can be used to express the magnetization of a material, such as human blood, in terms of blood flow NMR parameters, including the spin-spin relaxation time, spin-lattice relaxation time, blood flow velocity, gyromagnetic ratio, and radio frequency magnetic (RF) and static magnetic fields. In addition, the effects of relaxation, precession, and inhomogeneity of magnetic fields can all be explained using the Bloch NMR fluid flow equation (Awojoyogbe *et al.*, 2011). Mathematically, the solution to the Bloch NMR fluid flow time-dependent equation can be used to describe the NMR signal produced by blood proton spins as they move through human blood vessels. NMR signal, which is also known as free induction decay signal, is the time domain signal produced when the precessing atomic nuclei of a material, such as human blood, are disturbed by a short, strong radiofrequency (RF) pulse and then allowed to relax back to their equilibrium position in the presence of a static magnetic field (Carbajo *et al*., 2013; Carbajo & Neira, 2013; Webb, 2016). In addition, the NMR signal can be defined as the signal that the NMR machine detects, which represents the magnetization of the blood proton spins, which can be analyzed to extract information about the properties of the blood, such as its flow rate, oxygenation level, and volume. This information can be used in many potential applications, such as diagnosing and monitoring diseases (Carlier *et al*., 2006; Hoult, 1981; Lindon *et al.*, 2004; Theis & Meyer-Base, 2010).

Several studies have been carried out on the NMR signals and it applications (Chance *et al*., 1978; Heidari & Gobato, 2020; Horn *et al.*, 2000; Karseev *et al.*, 2015; Myazin & Davydov, 2018; Prance & Aydin, 2007; Torrey, 1956). Gale and Pierre (1995) looked at statistical differences in the aliphatic region of blood plasma to determine whether NMR signal analysis might be used to identify malignant cancer. Using a preprocessing methodology to remove unnecessary components, this study presents a novel approach for directly estimating nine components within the methyl and methylene peaks. They utilized an order reduction algorithm, Kumaresan and Tufts' modification, and Prony's technique to find the pertinent chemicals. Finally, their results showed a strong correlation. Fabry and San George (1983) examined the impact of magnetic susceptibility on NMR signals using a sample of red blood cells. They investigated these materials using NMR spectroscopy and then concluded that magnetic susceptibility had a considerable impact on the NMR signals, which could result in inaccurate data interpretation. Jenson and Chandra (2000) formulated a theory to explain how NMR signals in magnetically heterogeneous tissues react to a strong external magnetic field. The study showed that the decay times of single-spin echo, multiple-spin echo, and free induction decay signals increase with the strength of the external magnetic field. In order to validate their theoretical findings, they used numerical models with randomly distributed magnetic spheres. Gao *et al*. (1988) obtained an expression that defined mean image signal intensity in terms of flip angle, pulse sequence repetition interval (TR), and flow velocity by considering both laminar and plug flow models. Their results showed that the degree of flow enhancement is determined by the rate of approach to steady-state conditions. Endre *et al*. (1984) discovered that the mean field gradients for the extracellular and intracellular compartments were 0.89–2.09 G/cm and 0.25–1.98 G/cm, respectively. They concluded that spin-echo NMR may be utilized for monitoring changes in cell volume during trans-membrane molecule movement and that maintaining isotonicity in extracellular fluid during metabolic research is essential to prevent artifacts in metabolite concentration determination.

Recently, Rasheed and Usman (2023) theoretically formulated a nuclear magnetic resonance (NMR) signal, also known as free induction decay (FID) signal, from blood proton spins flowing along the human blood vessel by using the Bloch NMR fluid flow equation. However, it was observed that the Bloch NMR fluid flow equation did not account for certain NMR blood flow parameters, such as the applied magnetic field of the NMR spectrometer and the precessional frequency of the blood protons. In order to address this issue and generate more accurate NMR signals, this study aims to develop a novel NMR signal equation that includes these previously missing parameters by using the newly developed improved Bloch NMR fluid flow equation built by Rasheed *et al*. (2024). Additionally, the study intends to investigate the impact of relaxation time on the NMR signal generated from blood proton spins flowing through human blood vessels, using an improved Bloch NMR fluid flow equation and Laplace Transform method.

Improved Bloch NMR Fluid Flow Equation

The improved Bloch NMR fluid flow equation, which is a third-order linear partial differential equation as derived by Rasheed *et al*. (2024), is given as:

$$v^3T_1T_2^2\frac{\partial^3 M_y}{\partial x^3} + T_1T_2^2\frac{\partial^3 M_y}{\partial t^3} + 3v^2T_1T_2^2\frac{\partial^3 M_y}{\partial t\,\partial x^2} + 3vT_1T_2^2\frac{\partial^3 M_y}{\partial x\,\partial t^2} + (2v^2T_1T_2 + v^2T_2^2)\frac{\partial^2 M_y}{\partial x^2} + (T_2^2 + 2T_1T_2)\frac{\partial^2 M_y}{\partial t^2} + (4vT_1T_2 + 2vT_2^2)\frac{\partial^2 M_y}{\partial x\,\partial t} + (T_1 + 2T_2)\frac{\partial M_y}{\partial t} + (2vT_2 + 2vT_1)\frac{\partial M_y}{\partial x} + (T_1T_2\gamma^2B_1^2 + T_2^2\gamma^2B_0^2 + 1)M_y = \gamma T_2B_1M_0(\cos\omega t - 2T_2\gamma B_0\sin\omega t) \quad (1)$$

where M_y is the transverse magnetization of human blood spinning protons, γ is the gyromagnetic ratio of blood proton spins, B_0 is the static magnetic field, B_1 is the radio-frequency (RF) magnetic field, T_1 is the longitudinal or spin lattice relaxation time of blood proton spins, T_2 is the transverse or spin-spin relaxation time of blood proton spins, ω is the Larmor frequency of blood proton spins, and M_0 is the equilibrium magnetization of blood proton spins.

NMR Signal of Blood Proton Spins

The NMR signal, which is the transient state solution to the improved Bloch NMR fluid flow space-time dependent equation, can be defined as the solution that describes the time varying behavior of blood proton spins in human blood vessel in response to an external magnetic field. In order to obtain the NMR magnetization signal equation, one needs to divide both sides of Eq. (1) by $v^3T_1T_2^2$. Thus, we have:

$$\frac{\partial^3 M_y}{\partial x^3}+\frac{1}{v^3}\frac{\partial^3 M_y}{\partial t^3}+\frac{3}{v}\frac{\partial^3 M_y}{\partial t\,\partial x^2}+\frac{3}{v^2}\frac{\partial^3 M_y}{\partial x\,\partial t^2}+\left(\frac{2T_1+T_2}{vT_1T_2}\right)\frac{\partial^2 M_y}{\partial x^2}+\left(\frac{4T_1+2T_2}{v^2T_1T_2}\right)\frac{\partial^2 M_y}{\partial x\,\partial t}+\left(\frac{T_2+2T_1}{v^3T_1T_2}\right)\frac{\partial^2 M_y}{\partial t^2}+\left(\frac{T_1+2T_2}{v^3T_1T_2^2}\right)\frac{\partial M_y}{\partial t}+\left(\frac{T_1+2T_2}{v^2T_1T_2^2}\right)\frac{\partial M_y}{\partial x}+\left(\frac{T_1T_2\gamma^2B_1^2+T_2\gamma^2B_0^2+1}{v^3T_1T_2^2}\right)M_y=\frac{\gamma B_1M_0}{v^3T_1}\left[\frac{\cos\omega t}{T_2}-2\gamma B_0\sin\omega\, t\right] \quad (2)$$

When blood flows through human blood vessel and the flow does not vary spatially, this implies that the partial derivatives with respect to *x* and variable *x* present in the improved Bloch NMR fluid flow space-time dependent equation can be set to zero. This makes the improved Bloch NMR fluid flow space-time equation to change to improved Bloch NMR time dependent equation. Thus, one gets:

$$\frac{1}{v^3}\frac{d^3 M_y}{dt^3}+\left(\frac{T_2+2T_1}{v^3T_1T_2}\right)\frac{d^2 M_y}{dt^2}+\left(\frac{T_1+2T_2}{v^3T_1T_2^2}\right)\frac{dM_y}{dt}+\left(\frac{T_1T_2B_1^2+T_2\gamma^2B_0^2+1}{v^3T_1T_2^2}\right)M_y=\frac{\gamma B_1M_0}{v^3T_1}\left(\frac{\cos\omega t}{T_2}-2\gamma B_0\sin\omega\, t\right) \quad (3)$$

On multiplying both sides of Eq. (3) by v^3. One gets:

$$\frac{d^3 M_y}{dt^3}+\left(\frac{T_2+2T_1}{T_1T_2}\right)\frac{d^2 M_y}{dt^2}+\left(\frac{T_1+2T_2}{T_1T_2^2}\right)\frac{dM_y}{dt}+\left(\frac{T_1T_2B_1^2+T_2\gamma^2B_0^2+1}{T_1T_2^2}\right)M_y=\frac{\gamma B_1M_0}{T_1}\left(\frac{\cos\omega t}{T_2}-2\gamma B_0\sin\omega\, t\right) \quad (4)$$

Eq. (4) is a novel improved Bloch NMR fluid flow time dependent equation that has been developed in this research work. This equation describes the NMR magnetization signal generated from blood proton spins as they flow along the human vessels.

Let

$$f_1=\frac{T_2+2T_1}{T_1T_2} \quad (5)$$

$$f_2=\frac{T_1+2T_2}{T_1T_2^2} \quad (6)$$

$$f_3=\frac{T_1T_2\gamma^2B_1^2+T_2^2\gamma^2B_0^2+1}{T_1T_2^2} \quad (7)$$

$$f_4=\frac{\gamma B_1M_0}{T_1T_2} \quad (8)$$

$$f_5=\frac{2\gamma^2M_0B_0B_1}{T_1} \quad (9)$$

Plugging Eqs. (5), (6), (7), (8), and (9) into Eq. (4). One gets:

$$\frac{d^3 M_y}{dt^3}+f_1\frac{d^2 M_y}{dt^2}+f_2\frac{dM_y}{dt}+f_3M_y=f_4\cos\omega\, t+f_5\sin\omega\, t \quad (10)$$

Eq. (10) is a non-homogeneous linear differential equation of third order, which has been solved using the Laplace Transform method. This technique yielded a closed form solution, which is expressed as follows:

$$M_y(t)=2(f_{21}\cos\omega\, t+f_{22}\sin\omega\, t)+2(f_{27}\cos f_{16}\, t+f_{28}\sin f_{16}\, t)e^{f_{15}t}+f_{29}e^{s_3t} \quad (11)$$

Eq. (11) is the newly developed NMR signal equation which represents NMR signal generated from blood proton spins flowing along human blood vessels.

where the symbols s_3 and f_i (i = 6, 7,…,26) appearing in Eq. (11) have the following definitions:

$$f_6=kf_1 \quad (12)$$
$$f_7=kf_2+k\omega^2 \quad (13)$$
$$f_8=\omega^2f_1k+f_4 \quad (14)$$
$$f_9=\omega^2kf_2+f_5\omega \quad (15)$$
$$f_{10}=3f_2+3\omega^2 \quad (16)$$
$$f_{11}=2f_3+2f_1\omega^2 \quad (17)$$
$$f_{12}=\omega^2f_2 \quad (18)$$
$$f_{13}=\frac{3f_2-f_1^2}{3} \quad (19)$$
$$f_{14}=\frac{2f_1^3+27f_3-9f_1f_2}{27} \quad (20)$$
$$G=\frac{f_{14}}{2} \quad (21)$$
$$H=\sqrt{\frac{f_{13}}{27}+\frac{f_{14}}{4}} \quad (22)$$
$$s_3=(-G+H)^{\frac{1}{3}}+(-G-H)^{\frac{1}{3}}-f_1/3 \quad (23)$$
$$f_{15}=-\left(\frac{f_1+s_3}{2}\right) \quad (24)$$
$$f_{16}=\sqrt{\frac{3s_3^2+2f_1s_3+4f_2-f_1^2}{2}} \quad (25)$$
$$f_{17}=\omega^4k+f_9-f_7\omega^2 \quad (26)$$
$$f_{18}=f_6\omega^3-f_8\omega \quad (27)$$
$$f_{19}=5\omega^4+f_{12}-f_{10}\omega^2 \quad (28)$$
$$f_{20}=4f_1\omega^3-f_{11}\omega \quad (29)$$
$$f_{21}=\frac{f_{17}f_{19}+f_{18}f_{20}}{f_{19}^2+f_{20}^2} \quad (30)$$
$$f_{22}=\frac{f_{18}f_{19}-f_{17}f_{20}}{f_{19}^2+f_{20}^2} \quad (31)$$
$$f_{23}=k(f_{15}^4+f_{16}^4-6f_{15}^2f_{16}^2)+f_6(f_{15}^3-3f_{15}f_{16}^2)+f_7(f_{15}^2-f_{16}^2)+f_8f_{15}+f_9 \quad (32)$$
$$f_{24}=4kf_{15}f_{16}(f_{16}^2-f_{15}^2)+f_6f_{16}(f_{16}^2-3f_{15}^2)-f_{16}(2f_7f_{15}-f_8) \quad (33)$$
$$f_{25}=5(f_{15}^4+f_{16}^4-6f_{15}^2f_{16}^2)+4f_1(f_{15}^3-3f_{15}f_{16}^2)+f_{10}(f_{15}^2-f_{16}^2)+f_{11}f_{15}+f_{12} \quad (34)$$
$$f_{26}=20f_{16}(f_{15}f_{16}^2-f_{13}^3)+4f_1f_{16}(f_{16}^2-3f_{15}^2)-f_{16}(2f_{10}f_{15}-f_{11}) \quad (35)$$

MATERIALS AND METHODS

Materials

This study utilized MATAB and Origin pro software tools for data generation and simulation purposes. MATLAB was used to generate data from the NMR signal equation represented by Eq. (11), while Origin pro software tool was utilized to create visual representations of the data in the form of graphs.

Method

In this study, the Laplace Transform method was utilized to obtain the exact solution of the improved Bloch NMR fluid flow time dependent equation given by Eq. (11).

NMR Blood Flow Constants and Parameters used for Simulations

The solution of the improved Bloch NMR fluid flow time dependent equation, which represents the NMR magnetization signal generated from blood proton spins defined by Eq. (11), depends on various NMR blood flow parameters and constants. In order to simulate the effect of relaxation times of arterial, venous, and capillary blood samples on the NMR magnetization signal of blood proton spins, the values of NMR blood flow constants and parameters listed in Tables 1 and 2 were used.

Table 1: NMR Blood Flow Constants used for Simulations

NMR Blood Flow Constant	Value	Unit	Source
Static magnetic field (B_0)	1.5	*T*	Rasheed *et al.*, 2024
Radio frequency magnetic field (B_1)	1	*G*	Rasheed *et al.*, 2024
Gyromagnetic ratio (γ)	2.6752 x 10^8	$rads^{-1}T^{-1}$	Rasheed *et al.*, 2024
Length of human blood vessel (L)	0.2	*m*	Rasheed *et al.*, 2024
Equilibrium magnetization (M_0)	1	*A/m*	Rasheed *et al.*, 2024
Human blood vessel right hand side boundary condition ($M(0)$)	1	*A/m*	Rasheed *et al.*, 2024
Human blood vessel left hand side boundary condition ($M(L)$)	0	*A/m*	Rasheed *et al.*, 2024
Longitudinal relaxation time for arterial human blood (T_1)	1387	*ms*	Rasheed *et al.*, 2024
Longitudinal relaxation time for venous human blood (T_1)	1381	*ms*	Rasheed *et al.*, 2024
Longitudinal relaxation time for capillary human blood (T_1)	300	*ms*	Rasheed *et al.*, 2024
Blood flow velocity (v)	0.2	*m/s*	Rasheed *et al.*, 2024

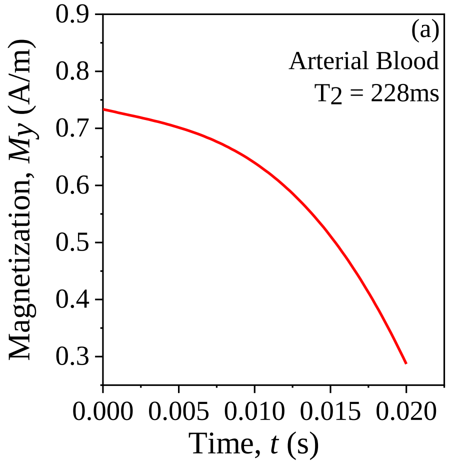
Table 2: NMR Blood Flow Parameters used for Simulations

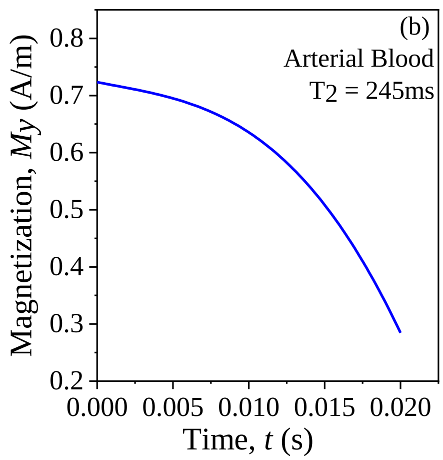
NMR Blood Flow Parameter	Value	Unit	Source
Transverse relaxation time for capillary human blood (T_2)	10-200	*ms*	Rasheed *et al.*, 2024
Transverse relaxation time for arterial human blood (T_2)	228-280	*ms*	Rasheed *et al.*, 2024
Transverse relaxation time for venous human blood (T_2)	158-204	*ms*	Rasheed *et al.*, 2024

RESULTS AND DISCUSSION

Results of Effect of Transverse Relaxation Time on NMR Signal Generated from Arterial Blood Proton Spins

The results of the effect of transverse relaxation time on the NMR signal generated from arterial blood spinning protons, as defined by equation (11), are graphically represented in Figure 1 with sub-plots (a-d). These figures illustrate how the NMR magnetization signal of arterial blood proton spins varies with time and decreases due to transverse relaxation, for various arterial blood flow parameter values T_2 = 228*ms*, 245*ms*, 262*ms*, and 279*ms* at T_1 = 1387*ms.*





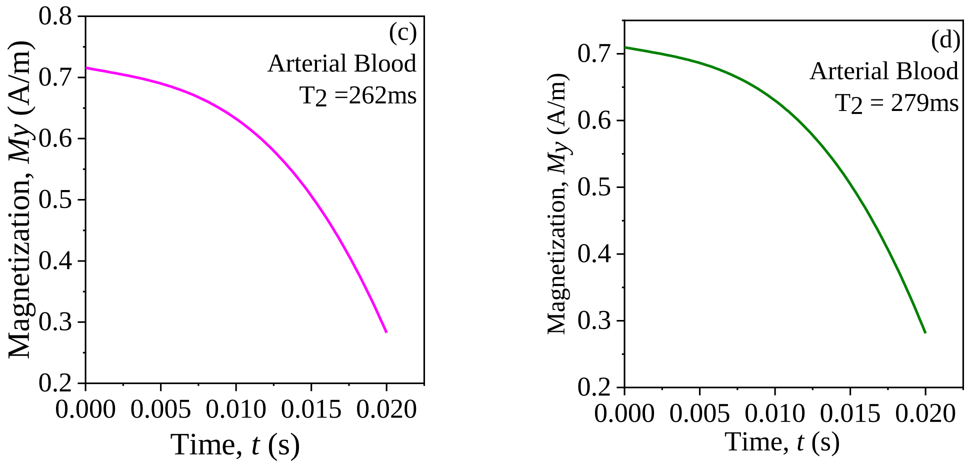


Figure 1: 2D plots of blood flow NMR magnetization signal defined by Eq. (11) for arterial blood with (a) T_2=228ms (b) T2 =245ms (c) T_2 =262ms (d) T_2 =279ms

Results of Effect of Transverse Relaxation Time on NMR Signal Generated from Venous Blood Proton Spins

Figure 2 with sub-plots (e-h) show graphical representations of NMR signal generated from venous blood spinning protons, as defined by Eq. (11), for venous blood flow parameter values T_2 = 158*ms*, 173*ms*, 188*ms*, and 203*ms* at T_1 = 1381*ms*. These results depict how the NMR magnetization signal of venous spinning protons changes with time and decreases due to transverse relaxation.

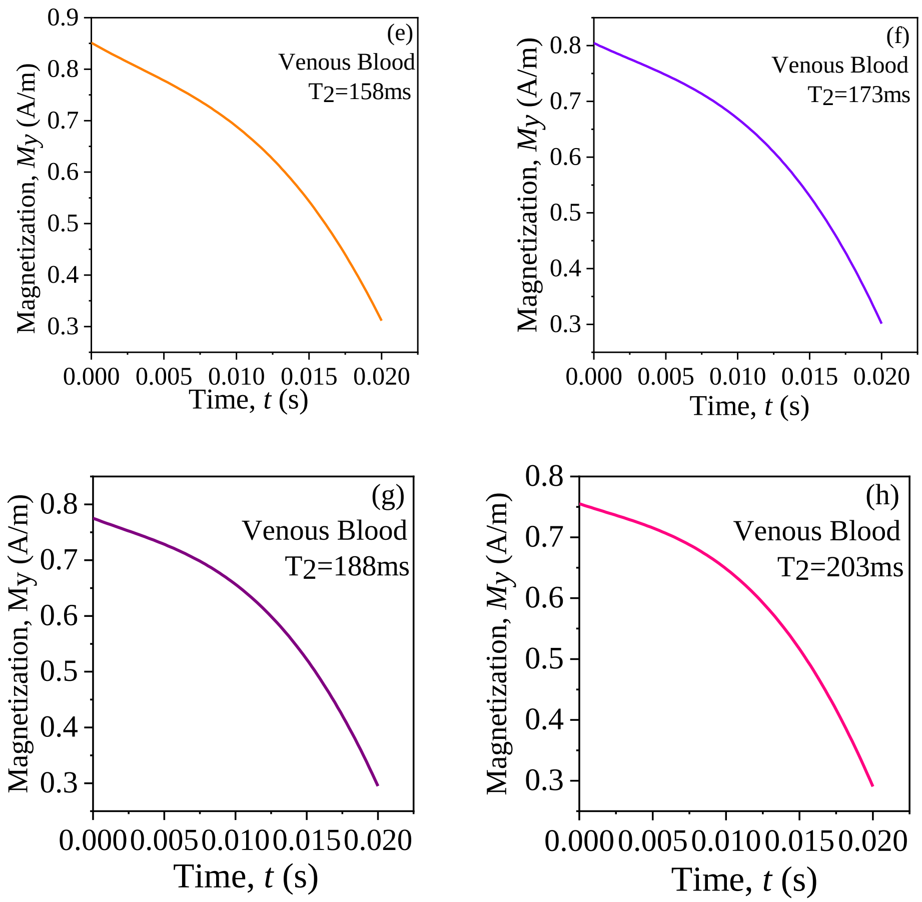


Figure 2: 2D plots of NMR signal defined by Eq. (11) for venous blood with (e) T_2=158ms (f) T2 =173ms (g) T_2 =188ms (h) T_2 =203ms

Results of Effect of Transverse Relaxation Time on NMR Signal Generated from Capillary Blood Proton Spins

The results of the effect of transverse relaxation times on the NMR signal generated from capillary blood spinning protons, defined by Eq. (11), are shown graphically in Figure 3 with sub-plots (i-l). These results illustrate temporal variation in NMR magnetization signal of capillary blood proton spins and decreases due to transverse relaxation, for a range of capillary blood flow parameter values T_2 = 10*ms*, 73*ms*, 136*ms*, and 199*ms* at T_1 = 300*ms*.

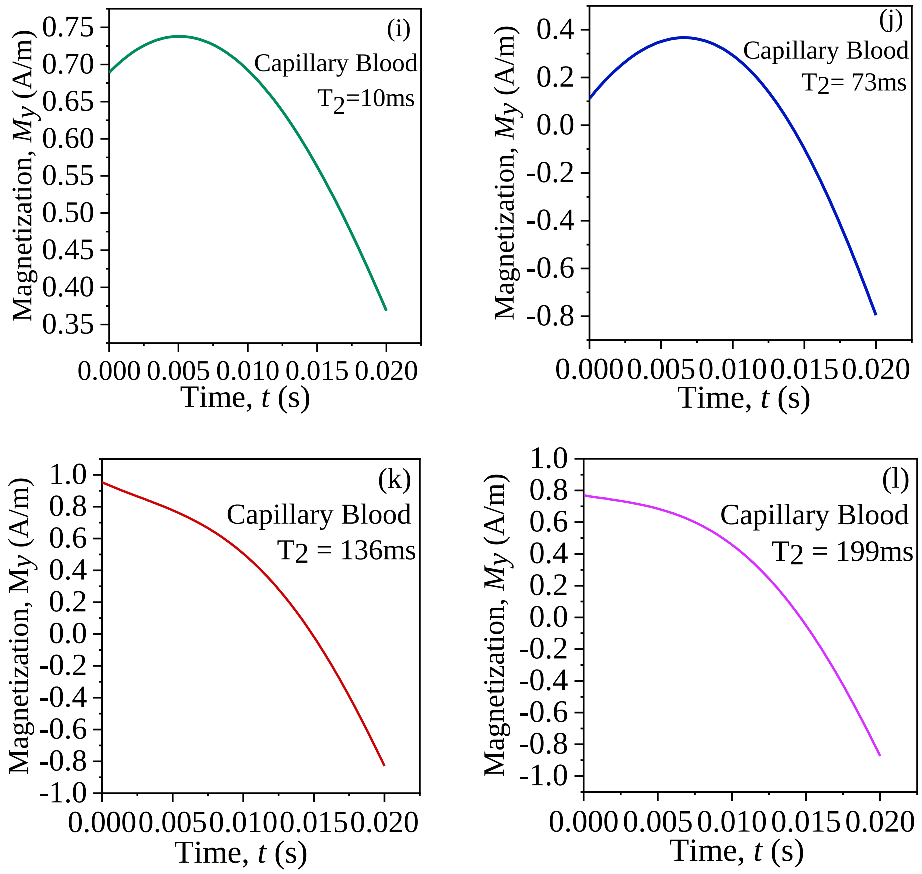


Figure 3: 2D plots of NMR signal defined by Eq. (11) for capillary blood with (i) T_2=10ms (j) T2 =73ms (k) T_2 =136ms (l) T_2 =199ms

Discussion

Figure 1 with sub-plots (a-d), Figure 2 with sub-plots (e-h), and Figure 3 with sub-plots (i-l) visually show the effect of varying the transverse relaxation time on the NMR signal generated from human blood proton spins in arterial, venous, and capillary blood, respectively. These figures illustrate the temporal variation in magnetization of human blood proton spins and indicated that, for varying transverse relaxation times of arterial, venous, and capillary blood, the magnetization of blood proton spins decreases as time increases. Thus, this means that the blood proton spins flowing along the human blood vessels loss the alignment of their magnetic moments due to the interactions between their neighboring spins, resulting in a progressive decrease in the NMR signal generated from blood proton spins. These interactions between the blood proton spins cause the magnetic moments to become more disordered over time. Furthermore, the results also imply that as blood flows through the human blood vessels, the blood-spinning protons are in phase, which means that they move in synch due to a decrease in the magnetization of blood-proton-spinning protons with increases in time. In addition, the results obtained are similar to the results of Abragam (1961), which show that the NMR signal, which is also called free induction decay (FID) signal, decreases as time increases. To conclude, the results do not agree with the result of Yusu *et al*. (2024), which shows that the NMR signal increases as time increases.

CONCLUSION

Conclusively, this study derived a novel NMR signal equation. The signal equation was utilized to investigate the effect of transverse relaxation times of arterial, venous, and capillary blood, respectively, on the NMR signal generated from blood proton spins flowing along the human blood vessels. The results of the simulations showed that the NMR signal decreases as time increases. This implies that the alignment of the magnetic moments of the blood proton spins is disrupted due to interactions between neighboring blood proton spins.

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