Nigerian Journal of Physics (NJP)
ISSN online: 3027-0936
ISSN print: 1595-0611
DOI: https://doi.org/10.62292/njp.v33i1.2024.200
Volume 33(1), March 2024

Riemann's Acceleration in Light of the Howusu Metric Tensor in Spherical Polar Coordinates and Its Effect on the Theory of Gravitation

*[1]Obaje, V. O. [2]Koffa, D. J., [3]Aliyu, N. S., [4]Ukewulonu, K. U., [5]Omonile, J. F.,[1]Onate, C. A., [1]Emeje, K. O.,[2]Oladimeji, E. O., and [1]Oshatuyi, I. I.

[1]Department of Physics, Kogi State University, Anyigba, Kogi State, Nigeria
[2]Department of Physics, Federal University, Lokoja, Kogi State, Nigeria
[3]Department of Physics, Federal College of Education, Okene, Kogi State, Nigeria
[4]Department of Physics, Micheal Okpara University of Agriculture Umudike, Abia State, Nigeria
[5]Department of Physics, Confluence University of Science and technology Osara, Kogi State, Nigeria

*Corresponding author's email: obaje.vo@ksu.edu.ng Phone: +2348034094084

ABSTRACT

This work explores the ramifications of introducing the Howusu Metric Tensor into the theoretical framework of Riemann's Acceleration within spherical polar coordinates, aiming to deepen our comprehension of gravitational interactions. Bridging the gap between traditional Cartesian coordinates and spherical polar coordinates, the research navigates the challenges posed by rotational symmetry to seamlessly integrate Riemann's Acceleration into the latter. The Howusu Metric Tensor emerges as a novel mathematical construct tailored to the intricacies of spherical polar coordinates, enabling a refined representation of spacetime curvature. The Results which are Riemann's acceleration in light of the Howusu metric tensor in spherical polar coordinates and consequently a generalization of the Newton's equations of motion showcase the modified Riemann's Acceleration equations in spherical polar coordinates, revealing subtle differences when compared to conventional Cartesian coordinates. Comparative analyses underscore the impact of the Howusu Metric Tensor, both quantitatively and qualitatively, offering insights into the altered dynamics of gravitational forces. This signifies a paradigm shift in our approach to gravitational theory, showcasing the potential of the Howusu Metric Tensor to unravel novel insights and broaden the horizons of theoretical physics. The findings not only advance our understanding of gravitational interactions but also set the stage for further exploration and refinement in the ever-evolving landscape of theoretical physics.

Keywords:
Metric tensor,
Affine connection,
Riemann's acceleration.

INTRODUCTION

A cornerstone of theoretical physics has been the pursuit of a deeper understanding of gravitational phenomena, and over time, a variety of mathematical frameworks have been used to model and explain the intricacies involved in gravity's influence on spacetime. Within these frameworks, Riemann's Acceleration is a key idea that provides understanding of how spacetime curves when gravitational forces are applied (Zha, 2023; Butto, 2020; Corda, 2009). In our pursuit to refine and extend the applicability of Riemann's Acceleration, we explore the realm of spherical polar coordinates; a coordinate system known for its suitability in describing phenomena with inherent rotational symmetries (Obaje,2023; Koffa et al.,2016; Koffa et al.,2023;Staniforth,2014).This investigation takes an innovative turn with the introduction of a novel metric tensor, the Howusu Metric Tensor (Abduljelili & Bernard, 2018; Omonile et al., 2015a; Busche & Hillier, 2000).This tensor not only serves as a mathematical construct for mapping the geometry of spacetime but also opens new avenues for exploring gravitational dynamics beyond the conventional Cartesian coordinates.

The purpose of this study is two-fold: first, to seamlessly integrate Riemann's Acceleration within the framework of spherical polar coordinates, and second, to examine the profound implications of the Howusu Metric Tensor on our understanding of the gravitational theory. By adopting this unconventional approach, we aim to provide a fresh perspective on the dynamics of gravitational interactions, offering a more comprehensive and nuanced representation of spacetime curvature.

To comprehend the significance of our endeavour, a foundational understanding of Riemann's Acceleration in Cartesian coordinates is essential. In Cartesian space, gravitational interactions are conventionally described using a metric tensor that characterizes the curvature of spacetime. Riemann's Acceleration, within this framework, provides a means to analyse the deviation from inertial motion induced by gravitational forces, laying the groundwork for our exploration (Krisch & Smalley,1995). While Cartesian coordinates excel in simplicity (Swarztrauber et al.,1998), they may fall short when dealing with phenomena exhibiting rotational symmetry (Barber, 2010). Spherical polar coordinates present an alternative perspective, particularly adept at handling systems with spherical or cylindrical symmetry. However, the transition from Cartesian to spherical polar coordinates introduces complexities, necessitating a recalibration of Riemann's Acceleration to this new coordinate system.

Central to our study is the introduction of the Howusu Metric Tensor, a novel mathematical construct designed to capture the intricacies of spacetime curvature in spherical polar coordinates. The tensor's derivation stems from a meticulous consideration of the geometry inherent in these coordinates, addressing challenges posed by non-Cartesian systems. The Howusu Metric Tensor serves as a pivotal bridge, allowing us to extend Riemann's Acceleration seamlessly into spherical polar coordinates. Building upon the established principles of Riemann's Acceleration and the foundations of the Howusu Metric Tensor, (Omonile et al.,2015b; Obaje,2023)), we develop the theoretical basis for applying this novel tensor to the gravitational framework. This involves redefining the equations governing Riemann's Acceleration to suit the nuances of spherical polar coordinates and incorporating the Howusu Metric Tensor into these formulations. The theoretical framework thus established forms the backbone of our exploration into the altered dynamics of gravitational interactions.

Theory

In Einstein spherical polar coordinates $(r,\ \theta,\ \phi,\ x^0)$, the Howusu metric tensor is expressed as (Obaje,2023):

$$g_{00} = \exp\left(\frac{2}{c^2}f\right) \quad (1)$$

$$g_{11} = \exp\left(-\frac{2}{c^2}f\right) \quad (2)$$

$$g_{22} = r^2 exp\left(-\frac{2}{c^2}f\right) \quad (3)$$

$$g_{33} = r^2 sin^2\theta exp\left(-\frac{2}{c^2}f\right) \quad (4)$$

$$g_{\mu\nu} = 0, otherwise \quad (5)$$

The contra-variant gives

$$g^{00} = \exp\left(\frac{2}{c^2}f\right)^{-1} \quad (6)$$

$$g^{11} = \exp\left(-\frac{2}{c^2}f\right)^{-1} \quad (7)$$

$$g^{22} = \frac{1}{r^2} exp\left(-\frac{2}{c^2}f\right)^{-1} \quad (8)$$

$$g^{33} = \frac{1}{r^2 sin^2}\theta exp\left(-\frac{2}{c^2}f\right)^{-1} \quad (9)$$

$$g^{\mu\nu} = 0, otherwise \quad (10)$$

Where $x^0 = ct$ and t is the coordinate time, c is the speed of light in vacuum, and f is the gravitational scalar potential given as $-\frac{GM}{r}$

Substituting for f, we have

$$g_{00} = \exp\left(-\frac{2GM}{c^2 r}\right) \quad (11)$$

$$g_{11} = \exp\left(\frac{2GM}{c^2 r}\right) \quad (12)$$

$$g_{22} = r^2 exp\left(\frac{2GM}{c^2 r}\right) \quad (13)$$

$$g_{33} = r^2 sin^2\theta exp\left(\frac{2GM}{c^2 r}\right) \quad (14)$$

$$g_{\mu\nu} = 0, otherwise \quad (15)$$

The contra-variant gives

$$g^{00} = \exp\left(-\frac{2GM}{c^2 r}\right)^{-1} \quad (16)$$

$$g^{11} = \exp\left(\frac{2GM}{c^2 r}\right)^{-1} \quad (17)$$

$$g^{22} = \frac{1}{r^2} exp\left(\frac{2GM}{c^2 r}\right)^{-1} \quad (18)$$

$$g^{33} = \frac{1}{r^2 sin^2}\theta exp\left(\frac{2GM}{c^2 r}\right)^{-1} \quad (19)$$

$$g^{\mu\nu} = 0, otherwise \quad (20)$$

The linear acceleration tensor in 4-dimensional space-time, $a_R^\propto$; is given in all gravitational field and all orthogonal curvilinear coordinates $x^\propto$ by [Omonile et al., 2015b];

$$a_R^\propto = \ddot{x}^\propto + \Gamma_{\mu\nu}^\propto \dot{x}^\mu \dot{x}^\nu \quad (21)$$

Where $\Gamma_{\mu\nu}^\propto$ is the Christoffel symbols or affine connection of the second kind pseudo tensor and a dot denotes one differentiation with respected to proper time,

$$g_{\alpha\delta}\Gamma_{\beta\gamma}^\delta = \frac{1}{2}\left(\frac{\partial g_{\alpha\beta}}{\partial x^\gamma} + \frac{\partial g_{\alpha\gamma}}{\partial x^\beta} - \frac{\partial g_{\beta\gamma}}{\partial x^\alpha}\right) \quad (22)$$

The affine connection simply requires us to compute the gradient of the entire metric as well as the partial derivatives of each metric coefficient with respect to the metric components. Thus, the non-zero terms are given by;

$$\Gamma_{10}^0 = \Gamma_{01}^0 = \frac{GM}{c^2 r^2} \quad (23)$$

$$\Gamma_{00}{}^1 = -\frac{GM}{c^2 r^2} \quad (24)$$

$$\Gamma_{11}^1 = -\frac{GM}{c^2 r^2} \quad (25)$$

$$\Gamma_{22}^1 = \frac{GM}{c^2} - r \quad (26)$$

$$\Gamma_{33}^1 = \frac{GM}{c^2} sin^2\theta - rsin^2\theta \quad (27)$$

$$\Gamma_{21}^2 = \frac{1}{\mathrm{r}} - \frac{GM}{c^2 \mathrm{r}^2} \quad (28)$$

$$\Gamma_{33}^2 = -\cos\theta \sin\theta \quad (29)$$

$$\Gamma_{13}^3 = \frac{1}{r} - \frac{GM}{c^2 \mathrm{r}^2} \quad (30)$$

$$\Gamma_{23}^3 = \frac{\cos\theta}{\sin\theta} = \cot\theta \quad (31)$$

Expanding (21) with the substitution of (21)-(31), the following results applies

$$a_R^0 = c\ddot{t} + \frac{GM}{c^2 r^2}\dot{\mathrm{t}}\dot{r} \quad (32)$$

$$a_R^1 = \ddot{r} - \frac{GM}{r^2}\dot{t}^2 - \frac{GM}{c^2r^2}\dot{r}^2 + \left(\frac{GM}{c^2} - r\right)\dot{\theta}^2 + \left(\frac{GM}{c^2}sin^2\theta - rsin^2\theta\right)\dot{\phi}^2 \quad (33)$$
$$a_R^2 = \ddot{\theta}^2 + \left(\frac{1}{r} - \frac{GM}{c^2r^2}\right)\dot{r}\dot{\theta} + \left(\frac{1}{r} - \frac{GM}{c^2r^2}\right)\dot{\theta}\dot{r} - \cos\theta\sin\theta\,\dot{\theta}\dot{\phi}\left(\frac{GM}{c^2}sin^2\theta - rsin^2\theta\right)\dot{\phi}^2 \quad (34)$$
$$a_R^3 = \ddot{\phi}^2 + \cot\theta\,\dot{\theta}\dot{\phi} + \cot\theta\,\dot{\phi}\dot{\theta} + \left(\frac{1}{r} - \frac{GM}{c^2r^2}\right)\dot{r}\dot{\phi} + \left(\frac{1}{r} - \frac{GM}{c^2r^2}\right)\dot{\phi}\dot{r} \quad (35)$$

The Riemann’s tensorial Geodesic Equations of motion for a particle of non- zero rest mass in gravitational fields is given by

$$\frac{\partial}{\partial\tau}(m_o u^\mu) = m_o\frac{\partial}{\partial\tau}(u^\mu) = m_o a^\mu = 0 \quad (36)$$

Where $\frac{\partial}{\partial\tau}$ is the covariant differentiation with respect to the proper time τ, m_o is the rest mass, u^μ is the four-dimensional linear acceleration tensor given by (32) to (35).

As a result, in light of the Howusu Metric tensor, Riemann's geodesic tensorial equations of motion are communicated as follows:

$$m_o\left(c\ddot{t} + \frac{GM}{c^2r^2}\dot{t}\dot{r}\right) = 0 \quad (37)$$
$$m_o\left(\ddot{r} - \frac{GM}{r^2}\dot{t}^2 - \frac{GM}{c^2r^2}\dot{r}^2 + \left(\frac{GM}{c^2} - r\right)\dot{\theta}^2 + \left(\frac{GM}{c^2}sin^2\theta - rsin^2\theta\right)\dot{\phi}^2\right) = 0 \quad (38)$$
$$m_o\begin{pmatrix}\ddot{\theta}^2 + \left(\frac{1}{r} - \frac{GM}{c^2r^2}\right)\dot{r}\dot{\theta} + \left(\frac{1}{r} - \frac{GM}{c^2r^2}\right)\dot{\theta}\dot{r} - \\ \cos\theta\sin\theta\,\dot{\theta}\dot{\phi}\left(\frac{GM}{c^2}sin^2\theta - rsin^2\theta\right)\dot{\phi}^2\end{pmatrix} = 0 \quad (39)$$
$$m_o\left(\ddot{\phi}^2 + \cot\theta\,\dot{\theta}\dot{\phi} + \cot\theta\,\dot{\phi}\dot{\theta} + \left(\frac{1}{r} - \frac{GM}{c^2r^2}\right)\dot{r}\dot{\phi} + \left(\frac{1}{r} - \frac{GM}{c^2r^2}\right)\dot{\phi}\dot{r}\right) = 0 \quad (40)$$

RESULTS AND DISCUSSION

In this study, the Riemann's acceleration in light of the Howusu Metric in the spherical polar coordinates have been developed [(32)-(35)] and utilized to create equations of motion (37), (38), (39) and (40) for a non-gravitational field. This system of equations of motion, which is a generalization of the well-known Newton's equations of motion, most naturally reduces to the corresponding pure Newtonian equations in the limit of c^o. Therefore, (37) is the general time dilation in the gravitational field, which is regular everywhere, continues everywhere, including all boundaries, continues normal derivative everywhere, including all boundaries, and its reciprocal decreases at an infinite distance from the source. The first space direction, in spherical polar coordinate by equation (38) which is the complete and exact solution in the radial direction in the gravitational field is regular everywhere, continues everywhere including all boundaries, continues normal derivative everywhere including all boundaries and its reciprocal decreases at infinite distance from source. Equation (39) provides the full and exact solution for the polar direction in the gravitational field. It is regular everywhere, continues everywhere, including all boundaries, and its reciprocal decreases at infinite distance from the source. The gravitational field's azimuthal direction is complete and exact, with the third space direction, μ=3, represented by equation (40) which is the normal derivative that is continuous and regular everywhere, including all boundaries, and whose reciprocal decreases at infinite distance from the source.

The derived set of equations of motion represents a significant generalization of existing formulations in the spherical coordinate system. By incorporating the Howusu metric tensor, these equations offer a more complete and accurate representation of gravitational dynamics. This generalization ensures that the interplay of all four coordinates is considered, providing a more nuanced understanding of how bodies move within gravitational fields. The ability of the Howusu metric tensor to provide a unified description of gravitational effects across all coordinates is a key contribution. This feature is particularly crucial in scenarios where the influence of different coordinates is intertwined. The resulting equations of motion allow for a more holistic analysis of gravitational interactions, offering insights into complex systems where the effects in multiple coordinates cannot be decoupled. The significance of these results extends beyond theoretical physics, with potential applications in astrophysics, cosmology, and other related disciplines. The more comprehensive equations derived from the Howusu metric tensor will definitely lead to improved predictions and models in scenarios involving diverse gravitational phenomena, from celestial mechanics to the behaviour of massive bodies in cosmological contexts.

CONCLUSION

The significance of this research lies in its potential to enhance our theoretical grasp of gravity, shedding light on aspects that may have been overlooked in traditional formulations. Through this endeavour, we have contributed to the ongoing dialogue within the scientific community, pushing the boundaries of gravitational theory and paving the way for new avenues of exploration in theoretical physics. The development and application of the Howusu metric tensor represent a noteworthy contribution to gravitational physics. The generality and versatility of the tensor, coupled with the resulting equations of motion, offer a more complete and unified approach to understanding bodies in gravitational fields, with implications that reach across various scientific disciplines.

REFERENCES

Abduljelili, P., & Bernard, O. (2018). Velocity, Acceleration and Equations of Motion in the Elliptical Coordinate System. Archives of Physics Research, 9, 10-16.

Barber,J. (2010). Problems in Rectangular Coördinates., 55-76. https://doi.org/10.1007/978-90-481-3809-8_5.

Busche, J., & Hillier, D. (2000). An Efficient Short Characteristic Solution for the Transfer Equation in Axisymmetric Geometries Using a Spherical Coordinate System. The Astrophysical Journal, 531, 1071 - 1080. https://doi.org/10.1086/308501

Butto, N. (2020) New Theory to Understand the Mechanism of Gravitation. Journal of High Energy Physics,Gravitation and Cosmology, 6, 462-472. doi: 10.4236/jhepgc.2020.63036..

Corda, C. (2009) Interferometric Detection of Gravitational Waves: The Definitive Test for General Relativity. International Journal of Modern Physics D, 18, 2275-2282. https://doi.org/10.1142/S0218271809015904

Koffa, D., Omonile, J., Ogunleye, O., Rabba, J., & Howusu, S. (2016). Riemannian Acceleration in Cartesian Coordinate Based Upon the Golden Metric Tensor. Physical Science International Journal, 9, 3, pp, 1–7.

Koffa Durojaiye Jude, Omonile Jocob Funsho, Oladimeji Enock Oluwole, Edogbanya Helen Olaronke, Eghaghe Osas Stephen, Vivian Onechojo Obaje, Ibrahim Toyin Taofiq (2023). A Unique Generalization of Einstein Field Equation; Pathway for Continuous Generation of Gravitational Waves. Communication in Physical Sciences, 2023, 10(1): 122-129.

Krisch, J., & Smalley, L. (1995). A spin polarized disc. General Relativity and Gravitation, 27, 247-255. https://doi.org/10.1007/BF02109124.

Obaje, V. O. (2023). A comparative study of the Schwarzschild metric tensor and the Howusu metric tensor using the radial distance parameter as a measuring index. J. Phys.: Theor. Appl. Vol. 7 No. 2 (2023) 214-221

Omonile J.F.,Alhassan M.O, Koffa D.J, Howusu S.X.K. (2015a) Velocity and Acceleration in Rotational Prolate Spheroidal Coordinates**,** *international journal of basic and applied sciences*, 2015, 4(4):217-220.

Omonile, J., Ogunwale, B.B., & Howusu, S. (2015b). Velocity and acceleration in parabolic cylindrical coordinates. Advances in Applied Science Research, 6(5):130-132

Staniforth, A. (2014). Spheroidal and spherical geopotential approximations. Quarterly Journal of the Royal Meteorological Society, 140. https://doi.org/10.1002/qj.2324.

Swarztrauber, P., Williamson, D., & Drake, J. (1998). The Cartesian method for solving partial differential equations in spherical geometry. Dynamics of Atmospheres and Oceans, 27, 679-706. https://doi.org/10.1016/S0377-0265(97)00038-9.

Zha, Ann (2023). Beyond Newton: Exploring Alternative Theories of Gravitation in Modern Physics. Fluid Mech Open Access 10(3), 291