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Combination-combination synchronization of chaotic fractional order systems

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ABSTRACT
The quest for simple and practically implementable synchronization control functions is the fundamental motive for this research work because most of the control functions for synchronization are bulky thereby leading to complexity in implementation and high cost. This research paper examines the combination-combination synchronization of chaotic fractional order systems of identical order evolving from different initial conditions via the backstepping nonlinear control technique with the aim of reducing the control function's complexity and cost. So, based on the stability theory of fractional order systems, stable synchronization are designed via a backstepping technique using chaotic fractional order Duffing and Aneodo as the paradigm. Numerical simulations are presented to confirm the feasibility of the analytical technique. The number of control functions has been sufficiently reduced compared to previous work hence reduction in control functions complexity and reduced cost of implementation. The outcome of this work may be useful to give a better understanding of the underlying dynamical behaviour among several interacting particles particularly in particle physics.

Keywords:
Synchronization,
Fractional order,
Control functions complexity.

INTRODUCTION

Fractional-order calculus which is a generalized form of the integer order calculus has been used to give a better description of the integer order calculus. The descriptive abilities of integer-order calculus can be traced to the work of Leibniz and Hospital in 1695. The integer-order calculus depends only on the local characteristics of a function, whereas, fractional order calculus possesses the heredity ability to store all information of the function in a certain time, which is also called memory property (Zhang&Shu, 2022; Lazarevi et al, 2014; Li, et al, 2013; Bickel&West, 1998; Machado&Galhano, 2012; Kilbas et al, 2006.) Mathematical models based on fractional order calculus can describe the dynamic behaviour of nonlinear dynamical systems more accurately than integer order calculus (Boulaaras et al, 2023; Ali et al, 2023; Fadila et al, 2023).

Due to the accurate descriptive power of fractional order calculus, it has been applied to models and used to analyze several nonlinear fractional order systems (Tang et al, 2022; Jan&Boulaaras, 2022; Tang et al, 2022). Mathematical modeling of fractional order nonlinear dynamical systems theory has found applications in various disciplines such as physics, chemistry, medicine, mathematics, biology, economics, engineering, and psychology (Valentim, 2021; Mitkowski et al, 2022; Harris&Garra, 2017). As a result of numerous applications of fractional order nonlinear dynamical systems, many research papers have been published on the dynamical behavior of coupled nonlinear fractional dynamical systems such as synchronization, tracking, control, chimera, bistability, multistability, and many more (Liu et al, 2021; Liu et al, 2016; Janarthanan et al, 2021; Hegazi et al, 2013).

Among these coupled fractional order nonlinear dynamical behaviours, synchronization is the most investigated due to its applications in information transmission and secure communication theory (Platas-Garza et al, 2021; Velamore et al, 2021; Alghamdi, 2021; Matignon, 1996). There are many synchronization types and schemes, such as, complete synchronization, projective synchronization, generalized synchronization, and combination synchronization. Similarly, different types of synchronization just listed have been investigated using different methods such as linear feedback, active control, backstepping, slide mode control, and others (Srivastava et al, 2014; Liu et al, 2014; He&Chen, 2014; Wang et al, 2012; Razminia&Baleanu, 2013). Given several applications of synchronization, a lot of synchronization techniques have been developed, such as, combination synchronization, hybrid combination synchronization, combination-combination synchronization, and many more as seen (Yadav et al, 2019; Ogunjo et al, 2019; Ogunjo et al, 2018; Khan&Nigar, 2020; Yadav et al, 2018; Singh et al, 2017; Ojo et al, 2022). However, most of the designed control functions are very complex and not cost-effective, hence, not very suitable for practical implementation. As a result, this research paper presents combination-combination synchronization of identical Duffing and Aneodo fractional-order chaotic systems with a minimal number of control functions compared to the previously designed control function. The optimization of these control functions will greatly simplify the complexity of the control function and save the cost of practical implementation.

Mathematical Model Fractional Order

Fractional order refers to a mathematical concept that involves using non-integer exponents or orders in equations. The Grunwald–Letnikov definition of fractional order systems, the fractional order derivative of order α can be written as (Petras 2011):

$$D_t^q f(t) = \lim_{h-0} \frac{1}{h^\alpha} \sum_{j=0}^{\infty} (-1)^j \binom{q}{j} f(t-jh)$$

Where $0 < q < 1$ and, t is the integration time, h is the time step.

The binomial coefficients can be written in terms of the Gamma function as:

$$\binom{q}{j} = \frac{r(q+1)}{r(j+1)r(q-j+1)}$$

The Riemann Liouville definition of fractional derivative is given as:

$$D_t^{-q} f(t) = \frac{1}{r(n-q)} \frac{d^n}{dt^n} \int_a^t \frac{f(T)}{(t-T)^{\propto+1}} dT$$

Where n = α. The Caputo fractional derivatives can be written as:

$$D_t^q f(t) = \frac{1}{r(n-q)} \int_a^t \frac{f^{(n)}(T)}{(t-T)^{q-n+1}} dT$$

Where, n-1 < q < n, T is the integration variable.

Fractional order calculus has applications in a wide range of fields, including physics, engineering,

Finance, and biology. It is a powerful tool that can help researchers to better understand and predict the behavior of complex systems.

Combination-combination synchronization for identical Duffing oscillators

The Duffing oscillator, named after George Duffing is a nonlinear second-order differential equation used to model certain damped and driven oscillators with a more complicated potential than in simple harmonic motion. The phase portrait of the fractional order chaotic Duffing oscillator is shown in Fig. 1.

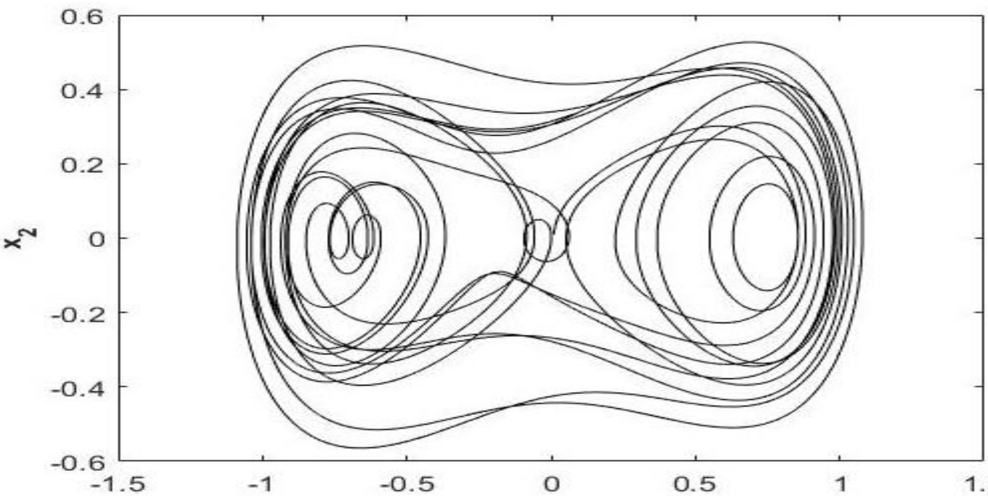

Figure 1: Chaotic attractor for fractional-order chaotic Duffing Oscillator for parameters a=0.01, α=-0.5, β=1, g=0.095, θ=0.79, $q = 0.98$

Now, for combination-combination synchronization, we shall consider two drive fractional order chaotic Duffing oscillators

$$D^q x_1 = x_2$$
$$D^q x_2 = -bx_2 + \alpha x_1 - \beta {x_1}^3 + g cos\theta t \qquad (1)$$
$$D^q x_3 = x_4$$
$$D^q x_4 = -bx_4 + \alpha x_3 - \beta {x_3}^3 + g cos\theta t$$

where $D^q x_i = \frac{d^q x_i}{dt}$ for $i = 1, 2, 3, 4$ is the fractional order differential of the variable x with respect to time.

The two response chaotic fractional order Duffing oscillators are

$$D^q y_1 = y_2 + U_1$$
$$D^q y_2 = -by_2 + \alpha y_1 - \beta {y_1}^3 + g cos\theta t + U_2 \qquad (2)$$

$D^q y_3 = y_4$
$D^q y_3 = -by_4 + \alpha y_3 - \beta {y_3}^3 + gcos\theta t + U_4$
Similarly, $D^q y_i = \frac{d^q y_i}{dt}$ for $i = 1, 2, 3, 4$ is the fractional order differential of the variable y with respect to time and U_1, U_2, U_3, U_4 are the controllers to be designed to ensure the realization of combination –combination synchronization.

The error variables for the drive and response systems are described mathematically as follows

$e_1 = y_1 + y_3 - (x_1 + x_3)$
$e_2 = y_2 + y_4 - (x_2 + x_4)$ (3)

Substitution of (1) and (2) into fractional order time differential of (3) yields

$D^q e_1 = e_2 + U_1 + U_3$
$D^q e_2 = -b(y_2 + y_4) + b(x_2 + x_4) + \alpha(y_1 + y_3) - \alpha(x_1 + x_3) + F_1 + U_2 + U_4$
$= -be_2 + \alpha e_1 + F_1 + U_2 + U_4$ (4)

where $F_1 = -\beta {y_1}^3 - \beta {y_3}^3 + \beta {x_1}^3 + \beta {x_3}^3$

So, the error dynamics can be summarized as follows

$D^q e_1 = e_2 + U_2 + U_4$
$D^q e_2 = -be_2 + e_1 + F_1 + U_2 + U_4$ (5)

Let $e_1 = q_1$, then, its fractional order time derivative is $D^\alpha e_1 = D^\alpha q_1$. Using Lyapunov function $V_1 = \frac{1}{2}{q_1}^2$, its fractional order time derivative is given as

$D^q V_1 = q_1 (e_2 + U_1 + U_3)$ (6)

If we write $e_2 = \alpha(q_1)$ as a virtual controller and $U_1 = U_3 = 0$, then $D^q V_1 = q_1(\alpha(q_1) + U_1 + U_3)$ which $\alpha(q_1) = -q_1$ in order for the equation to be negative definite. Then, we have

$D^q q_1 = q_1(-q_1) = -kq_1^2 < 0$ (7)

Which is negative definite.
The error between e_2 and α $(1(q_1)$ can be denoted by $q_2 = e_2 - \alpha_1(q_1)$.
Thus, we have the following (q_1, q_2) subsystems

$D^q q_2 = q_2 - kq_1^2$
$D^q q_2 = -b(q_2 - kq_1) + \alpha q_1 + F_1 + U_2 + U_4$ (8)

To stabilize systems (13), we choose a Lyapunov function, $V_2 = V_1 + \frac{1}{2}{q_2}^2$ whose fractional order time derivative is

$D^q V_2 = D^q V_1 + q_2 (D^q q_2)$
$= -k{q_1}^2 + q_2 (-bq_2 + bkq_1 + \alpha q_1 + F_1 + U_2 + U_4)$ (9)

If $U_2 + U_4 = -bkq_1 - \alpha q_1 - q_2(k - b) - F_1$. Therefore, $D^\alpha V_2 = -k{q_1}^2 - k{q_2}^2 < 0$ with $k > 0$. The errors are asymptotically stable hence, stable combination-combination synchronization is guaranteed. For simplicity, $U_2 = U_4 = U$, then,

$U = \frac{1}{2}[-bkq_1 - \alpha q_1 - q_2(k - b) - F_1]$ (10)

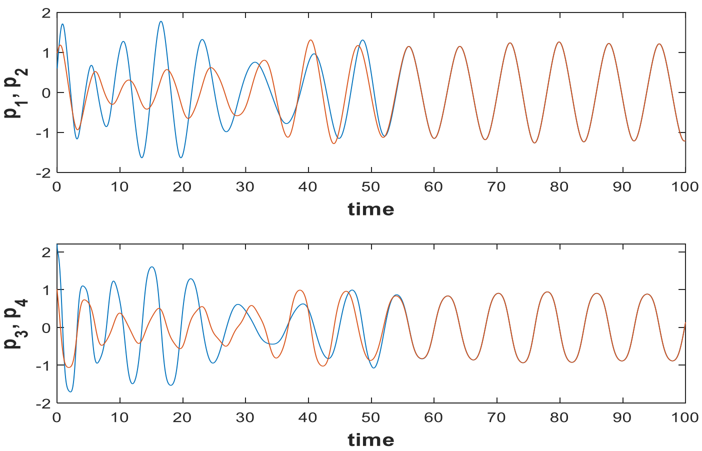

Figure 2: The dynamics of the state variables of the fractional order chaotic systems when the controllers are activated for $50 < t < 100$

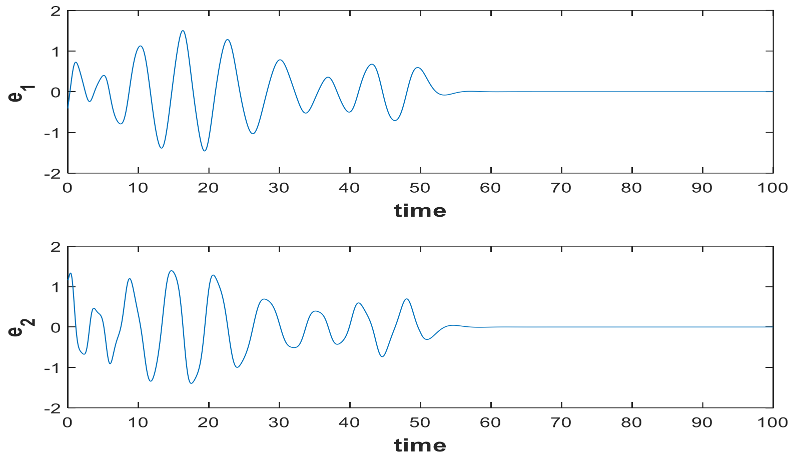

Figure 3: The dynamics of the error variables of the fractional order chaotic systems when the controllers are activated for $50 < t < 100$

Description of Combination synchronization of the Aneodo system

The mathematical model of the Aneodo system is described by the differential equation below

$$\frac{d^q p_1}{dt^q} = p_2$$
$$\frac{d^q p_2}{dt^q} = p_3$$
$$\frac{d^q p_3}{dt^q} = -\beta_1 p_1 - \beta_2 p_2 - \beta_3 p_3 + \beta_4 {p_1}^3 \qquad (15)$$

The phase portrait of the chaotic attractor of the Aneodo system is depicted in Fig. 4.

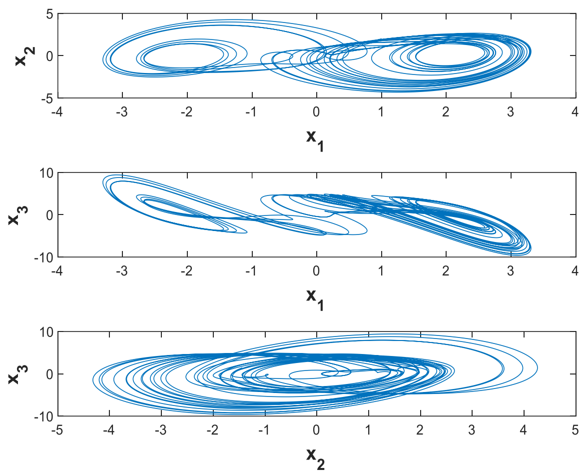

Figure 4: The phase portrait of chaotic attractor for fractional-order Aneodo system for parameters $\beta_1 = -5.5, \beta_2 = 3.5, \ \beta_3 = 1.0 \ \beta_4 = 1.0, b = 0.15, \alpha = 0.90$.

Combination synchronization shall be demonstrated by using four Aneodo systems starting from different initial conditions. The four Aneodo systems are represented by the following equations.

$$\frac{d^q x_1}{dt^q} = x_2$$
$$\frac{d^q x_2}{dt^q} = x_3$$
$$\frac{d^q x_3}{dt^q} = -\beta_1 x_1 - \beta_2 x_2 - \beta_3 x_3 + \beta_4 {x_1}^3 \qquad (16)$$

and

$$\frac{d^q y_1}{dt^q} = y_2$$
$$\frac{d^q y_2}{dt^q} = y_3$$
$$\frac{d^q y_3}{dt^q} = -\beta_1 y_1 - \beta_2 y_2 - \beta_3 y_3 + \beta_4 {y_1}^3 \qquad (17)$$

then the response systems

$$\frac{d^q z_1}{dt^q} = z_2$$
$$\frac{d^q z_2}{dt^q} = z_3$$

$\frac{d^q z_3}{dt^q} = -\beta_1 z_1 - \beta_2 z_2 - \beta_3 z_3 + \beta_4 {z_1}^3 + U_1$ (18)

and

$\frac{d^q w_1}{dt^q} = w_2$

$\frac{d^q w_2}{dt^q} = w_3$

$\frac{d^q w_3}{dt^q} = -\beta_1 w_1 - \beta_2 w_2 - \beta_3 w_3 + \beta_4 {w_1}^3 + U_2$ (19)

The error systems of (16) – (19) are

$e_1 = (z_1 + w_1) - (x_1 + y_1)$

$e_2 = (z_2 + w_2) - (x_2 + y_2)$

$e_3 = (z_3 + w_3) - (x_3 + y_3)$ (20)

Substitution of Equation (16) - (19) to the time derivative of Equation (20) yields

$\frac{d^q e_1}{dt^q} = e_2$ (21)

$\frac{d^q e_2}{dt^q} = e_3$ (22)

$\frac{d^q e_3}{dt^q} = -\beta_1(z_1 + w_1) + \beta_1(x_1 + y_1) - \beta_2(z_2 + w_2) + \beta_2(x_2 + y_2) - \beta_3(w_3 + z_3) + \beta_3(z_3 + y_3) + \beta_4 {w_1}^3 + \beta_4 {z_1}^2 - \beta_4 {x_1}^3 - \beta_4 {y_1}^3 + U_1 + U_2$

$= -\beta_1 e_1 - \beta_2 e_2 - \beta_3 e_2 + F_1 + U_1 + U_2$ (23)

where

$F_1 = \beta_4 {w_1}^3 + \beta_4 {z_1}^2 - \beta_4 {x_1}^3 - \beta_4 {y_1}^3$.

Let $e_1 = z_1$, then $\frac{d^q e_1}{dt^q} = \frac{d^q z_1}{dt^q} = e_2$ where (z_1) is regarded as virtual control input. To stabilize above system, we choose the Lyaponov function

$V_1 = \frac{{z_1}^2}{2}$ (24)

whose fractional order derivative is

$\frac{d^q V_1}{dt^q} = z_1(z_1) = z_1 \ \alpha_1(z_1)$ (25)

$If \ \alpha_1(z_1) \ is \ -z_1 \ then \ \frac{d^q U}{dt^q} = {z_1}^2 \leq 0.$ Then, Equation (25) is asymptotically stable since the virtual controller α_1 is estimative, the error difference between them is

$z_2 = e_2 - \alpha_1(z_1)$

so that

$z_2 = e_2 + z_1$ (26)

Then, we have the following subsystems $(z_1 z_2)$

$\frac{d^q z_1}{dt^q} = z_2 + z_1$

$\frac{d^q z_2}{dt^q} = e_3 + e_2 = e_3 + (z_2 - z_1)$

$\frac{d^q z_2}{dt^q} = e_3 + z_2 - z_1$ (27)

Where

$e_3 = \alpha_2(z_1, z_2)$ is regarded as virtual controller. We shall stabilize the system (27) using Lyapunov function

$V_2 = V_1 + \frac{{z_2}^2}{2}$ (28)

Whose derivative is

$\frac{d^q V_2}{dt^q} = \frac{d^q V_1}{dt^q} + z_2(\frac{d^q z_2}{dt^q})$

$= -{z_1}^2 + z_2(\alpha_3 + z_2 - z_1)$ (29)

If $\alpha_3 = z_1 - 2{z_2}^2$ then, $\frac{d^q V_2}{dt^q} = -{z_2}^2 - {z_2}^2 \leq 0$ then, the subsystem (27) asymptotically stable. Similarly, the error variable between α_2 and e_2 as $z_3 = e_3 - \alpha_3$ that is $e_3 = z_3 + \alpha_3$. Then we can denote the following (z_1, z_2, z_3)sub systems as

$\frac{d^q z_1}{dt^q} = z_2 + z_1$

$\frac{d^q z_2}{dt^q} = z_3 + z_1 - 2z_2 + z_2 - z_1$

$\frac{d^q z_3}{dt^q} = -\beta_1 z_1 - \beta_2(z_2 - z_1) - \beta_3(z_3 + z_1 - 2z_2) + F1 + U_1 + U_2$ (30)

To stabilize the (z_1, z_2, z_3), we choose the Lyapunov function of V_3 as

$V_3 = V_2 + \frac{{z_3}^2}{2}$ whose derivative is

$\frac{d^q v_3}{dt^q} = \frac{dv_2}{dt^q} + z_3 + \frac{dz_3}{dt^q}$ (31)

By appropriate substitutions

$\frac{d^q v^3}{dt^q} = -{z_1}^2 - {z_2}^2 - {\beta_3}^2 \leq 0 \ \ if$

$U_1 + U_2 = \beta_1 z_1 + \beta_2(z_2 - z_1) + \beta_3(z_1 - 2z_2) - F_1 - kz_3$

For simplicity, let $U_1 = U_2 = U$

then, $U = \frac{1}{2}(\beta_1 z_1 + \beta_2(z_2 - z_1) + \beta_3(z_1 - 2z_2) - F_1 - kz_3)$ (32)

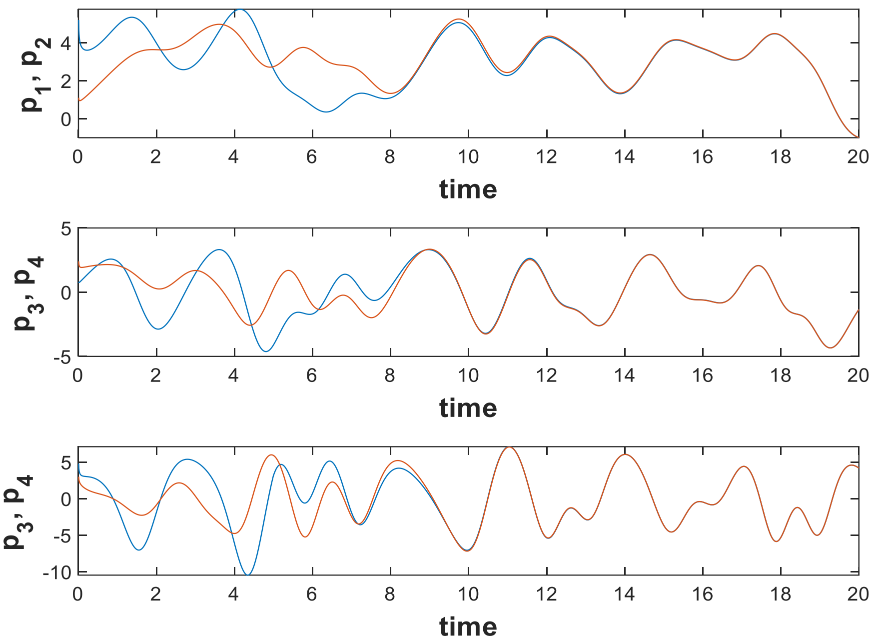

Figure 5: The dynamics of the state variables of the fractional order chaotic Aneodo when the controllers are activated for $10 < t < 20$

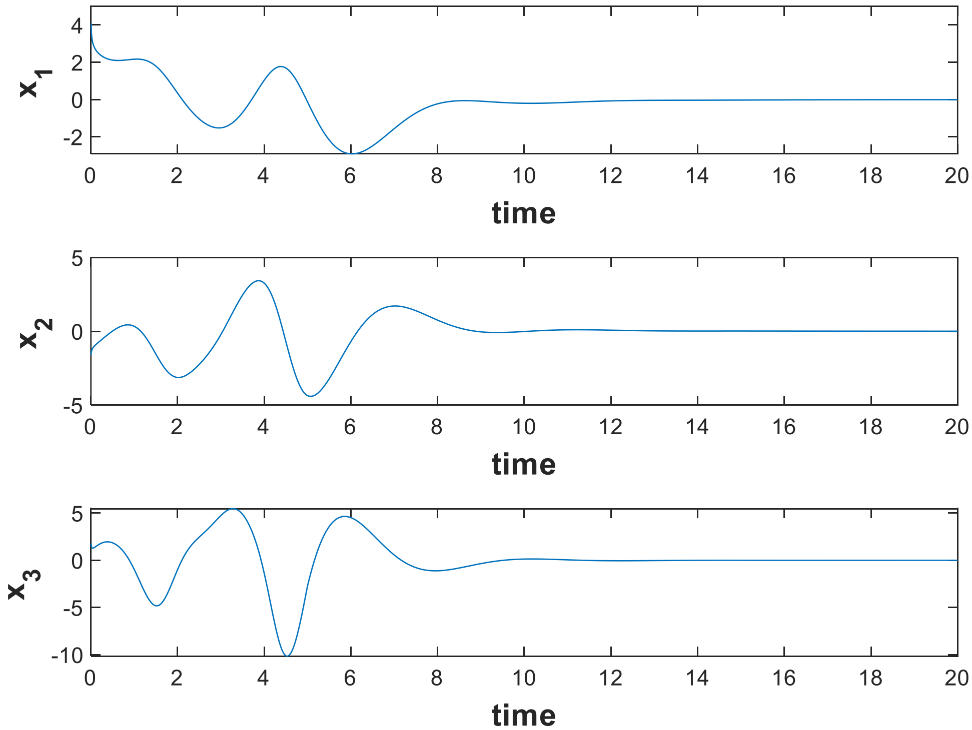

Figure 6: The dynamics of the error variables of the fractional order chaotic Aneodo when the controllers are activated for $10 < t < 20$

RESULTS AND DISCUSSION

The equations (1), (2), and (10) are solved numerically through the Runge-Kutta algorithm built-in MATLAB in order to authenticate the effectiveness of the derived controller in equation (10). The parameters values a=0.01, a=-0.5, β=1, g=0.095, θ=0.79, $\alpha = 0.98$ with the initial conditions (0.01, 0.01, 1, 1, 0.5, 2, 0.1, 0.2) in this numerical simulation. The result shown in Fig. 2

shows that the state variables moved in different directions before the controllers were applied. Immediately, the controllers are applied between $50 < t < 100$, and the state variables moved with an identical trajectory. The achievement of the trajectory shows clear evidence of synchronization. Similarly, another piece of evidence of synchronization is depicted in Fig. 3. Where the error variables are reduced to zero when the controllers are applied between $50 < t < 100$. This also confirms the synchronization of the systems.
Similarly, Numerical simulation was carried out by solving equations (16)-(19) with control defined in equation (32). The initial conditions of the fractional order Aneodo system are chosen as (0.1 0.4 1 1 2 2 2 0.2 4 3.2 0.6 0.8) with the following parameters $\beta_1 = -5.5, \beta_2 = 3.5,\ \beta_3 = 1.0\ \beta_4 = 1.0, b = 0.15, \alpha = 0.90$ in the chaotic region to ensure chaotic dynamics. The simulation result in Fig.5. shows that the state variable of the Aneodo systems moved with trajectories before the control functions were applied. Immediately the control functions were activated for $10 < t < 20$, the state variables begin to move with common trajectory. The identical dynamics achieved by the state variables after the application of the control functions show clear evidence of synchronization. Moreover, Fig.6. shows the dynamics of the error systems before and after the activation of the control functions for $10 < t < 20$. As shown in Fig.6, the error systems moved chaotically with time when the control functions were deactivated for $0 < t < 10$. When the control functions were activated for $10 < t < 20$ the error systems became zero which shows that the system synchronized. The implication of this is that after the application of the control functions the difference in their trajectories is zero. This indicates that combination-combination synchronization has been achieved with a minimal number of control functions.

CONCLUSION

This research shows the possibilities of achieving synchronization control in a combination-combination synchronization scheme through a minimal number of control functions. These optimized synchronization control functions give a bright hope for practical implementation with the advantages of simplicity and cost-effectiveness. Numerical simulations presented confirm the effectiveness of the analytically derived control functions. In particle physics, the obtained results could be helpful in giving a better understanding and insight into different synchronization patterns and interactions among different particles. Meanwhile, research is still going on to obtain more robust and simplified control functions with better cost-effectiveness.

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