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Synchronization Metric for Sinusoidally Coupled Periodically Modulated Josephson Junctions

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ABSTRACT

Synchronization control of coupled dynamical systems with bounded control functions in spite of advantages in the optimization of control functions has not been adequately explored. In order to further demonstrate the advantage of this control technique, this paper presents the effect of sinusoidal coupling on two periodically modulated chaotic Josephson junctions evolving from different initial conditions. A sufficient analytical criterion for the determination of the coupling threshold that leads to common dynamical behaviour called synchronization was derived. The derived analytical criterion is applied to sinusoidally coupled periodically modulated Josephson junctions within two dynamical regimes G_1 and G_2 to illustrate the effectiveness of the analytical criterion. Numerical simulations of the analytical result show the achievement of stable synchronization. The result can be applied to determine the level and the strength of interaction between several particles or objects interacting on different topological configurations that can result in different dynamical behaviours.

Keywords:
Josephson junctions,
Sinusoidal coupling,
Sylvester's criterion,
Synchronization,
Dynamical Regimes,
Coupling threshold.

INTRODUCTION

The studies of dynamical systems have given a better insight into the understanding of behavioural patterns of several natural and artificial systems such as biological, physical, chemical and socio-economical systems (Rita Akter 2019). Several phenomena such as synchronization, multistability, basin boundary crises, chimeras, and, different pattern formations have been discovered from interaction of two or more dynamical systems (Wen & Lu 2021). Synchronization is the most prominent among these dynamical behaviours exhibited by coupled dynamical systems and this is due to its applications in information communication, biological and physical systems (Ojo, Olusola & Njah 2013). Several types of synchronization such as complete synchronization, measure synchronization, anti-synchronization, generalized synchronization, projective synchronization, and many others have been investigated (Ayotunde, Kayode, Uchechukwu, And Njah 2014). Also, many nonlinear and linear synchronization methods have been developed in search of the most effective synchronization method. Some of the developed methods are linear coupling, cyclic coupling, backstepping, active control, optimal control, finite time and others (Jianping, Xiaofeng & Shuhui 2007) in the reference therein.

Meanwhile, most of the developed nonlinear control methods are very effective for synchronization of identical and non-identical dynamical systems. However, it is difficult to practically implement the control functions based nonlinear control methods: a result of many nonlinear terms involved in the control functions. In the case of linear feedback, though easy to implement practically but most of the designed linear feedback controllers are static in nature. These static controllers do not depict the true nature of the real world. Since dynamical problem can be best solved by dynamical solution, hence, dynamic coupling with bounded function is preferred. This research work is inspire based on the importance of bounded dynamic coupling and the dearth of research papers in this direction. The aim of this paper is to investigate synchronization in sinusoidally periodically modulated Josephson junctions

Brief description of periodically modulated Josephson junction

The equation for the periodically modulated Josephson junction described by Wu and Li (2007) is given by second order non-autonomous second order differential equation.

$\ddot{\phi} = -[1 + \xi\cos(\Omega t + \Theta)]\sin\phi + \rho_0 - \delta\dot{\phi} + \gamma\cos\omega t$ (1)

where ϕ is the phase difference between quantum mechanical wave functions of the two separated superconductors of the junction, $\xi\cos(\Omega t + \Theta)$ is the modulated terms with amplitude ξ, phase angle Θ,and frequency Ω, ρ_0 is the DC bias, δ is the damping parameter, and γ and ω are the amplitude and frequency of the RF-current, respectively. The sequence of periods-doubling route to chaos, as ξ is progressively increased, was reported by Ojo, Njah & Adebayo (2011). Fig. 1 shows a chaotic attractor for ξ = 2.462 arising from tori-doubling bifurcation.

Equation (1) can be represented as

$\dot{x_1} = x_2$

$\dot{x}_2 = -[1 + \xi\cos(\Omega t + \Theta)]\sin x_1 + \rho_0 - \delta x_2 + \gamma\cos(\omega t)$ (2)

where $\ddot{\phi} = \dot{x}_1$ and $\dot{\phi} = x_1$

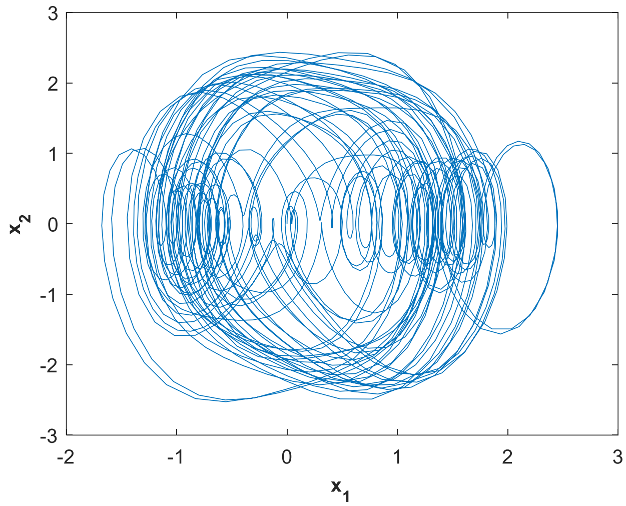

Figure 1: The phase space portrait of the chaotic attractor of the periodical excited Josephson junction with parameters $\Omega, \Theta = 0.1, \rho_0 = 0.1, \delta = 0.1, \gamma = 2, \omega = 3, \xi = 2.462$

Using vector notation $x = (x_1, x_2)^T \epsilon\, \boldsymbol{R}^2$ then, Equation (2) becomes

$\dot{X} = AX + B(x) + C(t)$ (3)

where

$$A = \begin{pmatrix} 0 & 1 \\ 0 & -\delta \end{pmatrix}, B(x) = \begin{pmatrix} 0 \\ -\sin x_1 - \xi\sin x_1\cos(\Omega t + \Theta) \end{pmatrix}\ C(t) = \begin{pmatrix} 0 \\ \rho_0 + \gamma\cos(\omega t) \end{pmatrix}$$

$$\begin{pmatrix} -k_1 S_1(t) & q(t) \\ 1 & -(\delta + k_2 S_2(t)) \end{pmatrix}\begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}\begin{pmatrix} 0 \\ \rho_0 + \gamma\cos(\omega t) \end{pmatrix},$$

$$\begin{pmatrix} -P_1 k_1 S_1(t) + 0 & 0 + -P_2 q(t) \\ P_1 + 0 & -P_2(\delta + k_2 S_2(t)) \end{pmatrix}$$

$P_1 S_1(t) k_1 > 0, 4P_1 P_2$

$P_1 \frac{2\sqrt{2}}{3\pi}.$

$k_1 > 0$

Design of synchronization threshold for the sinusoidal coupled periodically modulated Josephson junctions

A drive-response synchronization scheme for two identical Periodically Modulated Josephson Junction coupled via a sinusoidal state error feedback controller is constructed as follows:

Drive System:

$Ax + B(x) + C(t) = \dot{x}$ (4)

Response system:

$Ay + B(y) + C(t) + u(t) = \dot{y}$ (5)

Controller

$: u(t) = [k_1 Sin(x_1 - y_1),\ k_2 Sin(x_2 - y_2)]^T$ (6)

where $y = (y_1, y_2)^T$, T means transpose, and k_1 and k_2 are coefficients of coupling variables

Defining error variable

$e = x - y$ or $(e_1, e_2) = (x_1 - y_1, x_2 - y_2)$, we can obtain an error dynamical system

$$\dot{e} = A(x-y) - u(t) + B(x) - B(y)$$
$$\dot{e} = (A - kt) + N(t) \quad (7)$$

With

$$k(t) = \begin{pmatrix} k_1S_1(t) & 0 \\ 0 & k_2S_2(t) \end{pmatrix},$$
$$S_1(t) = \frac{Sin(x_1-y_1)}{x_1-y_1},\ S_2(t) = \frac{Sin(x_2-y_2)}{x_2-y_2}$$
$$N(t) = \begin{pmatrix} 0 & 0 \\ q(t) & 0 \end{pmatrix}\ q(t) = \frac{-(Sin\,x_1 - Sin\,y_1) - \xi\cos(\Omega t + \Theta)(Sin\,x_1 - Sin\,y_1)}{x_1-y_1} \quad (8)$$

We aim at choosing suitable coupling coefficients k_1, k_2 such that $x(t)$ and $y(t)$ satisfies the condition

$$\lim_{t\to\infty}\|x(t)-y(t)\| = \lim_{t\to\infty}\|e(t)\| = 0 \quad (9)$$

Such the error between the drive and the response systems asymptotically tends to zero after the activation of the coupling variables at appropriate coupling strength. Using stability theory, synchronization of chaos in systems Equation (4) and Equation (5) in line with Equation (9) is equivalent to the asymptotic stability of the error system (7) at the origin $e = 0$. Note that $\|x(t)-y(t)\| = \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2}$ denotes the Euclidean norm of vectors.

Taking a quadratic Lyapunov function $V(e) = e^TPe$ with P a symmetric positive definite. Constant matrix, where the derivative of $V(e)$ with respect to time along the trajectory of system (7) is $\dot{V}(e) = \dot{e}^TPe + e^TP\dot{e}$

$$\dot{V}(e) = e^TP(A - k(t) + N(t)) + \left(A - k(t) + N(t)\right)^T P]e \quad (10)$$

By the Lyapunov stability theorem for linear time-varying system, a sufficient condition that the error system (7) in asymptotically stable at the origin is given by

$$Q(t) = P(A - k(t) + N(t)) + \left(A - k(t) + N(t)\right)^T P \quad (11)$$

This is negative definite and denoted by

$$Q(t) < 0 \quad (12)$$

Now

$$A - k(t) + N(t) = \begin{pmatrix} 0 & 1 \\ 0 & -\delta \end{pmatrix} - \begin{pmatrix} k_1S_1(t) & 0 \\ 0 & k_2S_2(t) \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ q(t) & 0 \end{pmatrix}$$
$$A - k(t) + N(t) = \begin{pmatrix} -k_1S_1(t) & 1 \\ q(t) & -\left(\delta + k_2S_2(t)\right) \end{pmatrix}$$

For simplicity, we choose

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}$$

Therefore,

$$Q(t) = \begin{pmatrix} -2P_1k_1S_1(t) & P_1 + P_2q(t) \\ P_1 + P_2q(t) & -2P_2\left(\delta + k_2S_2(t)\right) \end{pmatrix} \quad (13)$$

By Sylvester's Criterion, $Q(t) < 0$ is equivalent to the following inequalities:

$$P_1k_1S_1(t) > 0, \quad 4P_1P_2k_1S_1(t)\left(\delta + k_2S_2(t)\right) > \left(P_1 + P_2q(t)\right)^2 \quad (14)$$

Note that $S_1(t) > 0$ and $S_2(t) > 0$ if (x_1, x_2) and (y_1, y_2) are limited in the region $R\{|x_1 - y_1| < \pi, |x_2 - y_2| < \pi\}$, So, error system (7) is asymptotically stable at the origin in the region R.if the inequality (14) is satisfied.

Determination of coupling threshold for bidirectional coupling

In this section, algebraic criterion for bidirectional coupled Josephson junctions with coupling parameters from two different bounded dynamical regimes G_1 and G_2

Case 1: Determination of coupling threshold using the algebraic criteria G_1

In order to get an easily verified algebraic condition, we further restrict the variables in the subregion $G_1 = \left\{|x_1 - y_1| \le \frac{3\pi}{4}, |x_2 - y_2| \le \frac{3\pi}{4}\right\}$ then, we have $\frac{2\sqrt{2}}{3\pi} \le S_1(t) \le 1$ and $\frac{2\sqrt{2}}{3\pi} \le S_2(t) \le 1$

Now a simple algebraic sufficient criterion for synchronizing the system (4) and (5) can be obtained from (14) as

$$k_1 > 0 \quad (15a)$$
$$4P_1P_2k_1\frac{2\sqrt{2}}{3\pi}\left(k_2\frac{2\sqrt{2}}{3\pi} + \delta\right) > \left(P_1 + P_2q(t)\right)^2$$
$$k_2 > \frac{9\pi^2\left(P_1 + P_2(1+|\xi|)\right)^2}{32P_1P_2k_1} - \frac{\delta 3\pi}{2\sqrt{2}} \quad (15b)$$

Deducing from lemma (Wu, Kai & Wang 2006), the inequality $|q(t)| < 1 + |\xi|$.

The synchronization criterion obtained here only renders a sufficient but not necessary condition. It is natural to expect that a sharper criterion can provide more choices of the coupling coefficients. To this end, we can minimize the lower bound of k_2 in inequality (15b) by choosing

$P = diag\{(1+|\xi|)P_2, P_2\}$

Now substituting $P = diag\{(1+|\xi|)P_2, P_2\}$ in (15) we have

$$k_1 > 0 \quad (16a)$$
$$k_2 > \frac{9\pi^2[(1+|\xi|)P_2 + P_2(1+|\xi|)]^2}{32(1+|\xi|)P_2^2k_1} - \frac{3\pi\delta}{2\sqrt{2}}$$
$$k_2 > \frac{9\pi^2[2P_2(1+|\xi|)]^2}{32(1+|\xi|)P_2^2k_1} - \frac{3\pi\delta}{2\sqrt{2}}$$
$$k_2 > \frac{9\pi^2(1+|\xi|)}{8k_1} - \frac{3\pi\delta}{2\sqrt{2}} \quad (16b)$$

Case 2: Determination of coupling threshold using the algebraic criteria G_2

$G_2 = \left\{|x_1 - y_1| \le \frac{\pi}{2}, |x_2 - y_2| \le \frac{3\pi}{4}\right\}$. Then we have $\frac{2}{\pi} \le S_1(t) \le 1$ and $\frac{2\sqrt{2}}{3\pi} \le S_2(t) \le 1$

Using G_1in Equation (14) yields

$P_1 k_1 \left(\frac{2}{\pi}\right) > 0$

$\Rightarrow k_{1,} > 0$ (17a)

$k_2 > \frac{3\pi^2\beta}{16\sqrt{2}P_1P_2k_1} - \frac{3\pi\delta}{2\sqrt{2}}$ $\left(\beta = \left(P_1 + P_2(1 + |\xi|)\right)^2\right)$ (17b)

To obtain a sharper criterion, we minimize the lower bound of k_2 in inequality (17b) by choosing $P = diag\{(1 + |\xi|)P_2, P_2\}$

Now substituting $P = diag\{(1 + |\xi|)P_2, P_2\}$ in (22), we obtain

$k_2 > \frac{3\pi^2(1+|\xi|)}{4\sqrt{2}k_1} - \frac{3\pi\delta}{2\sqrt{2}}$ (18)

Please note that numerical simulation result is only presented for case 2 since the result are similar. Now the numerical solution Equations (4) and (5) with activation of controllers defined in Equation (6) numerically using condition in equation (17a) and (18) for $100 \le t \le 200$ using inbuilt ODE45 solver in MATLAB. The results obtained is as shown in Figure (2) and Figure (3).

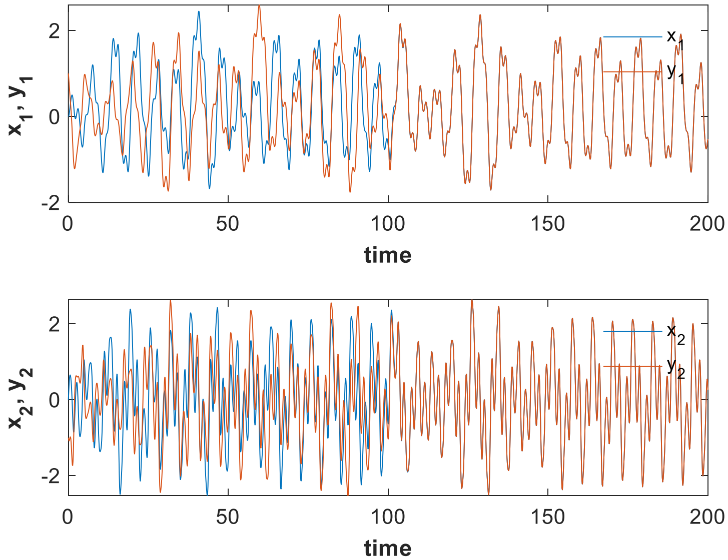

Figure 2: The dynamics of the state variables of the drive and response variable when the control functions are activated for $100 \le t \le 200$

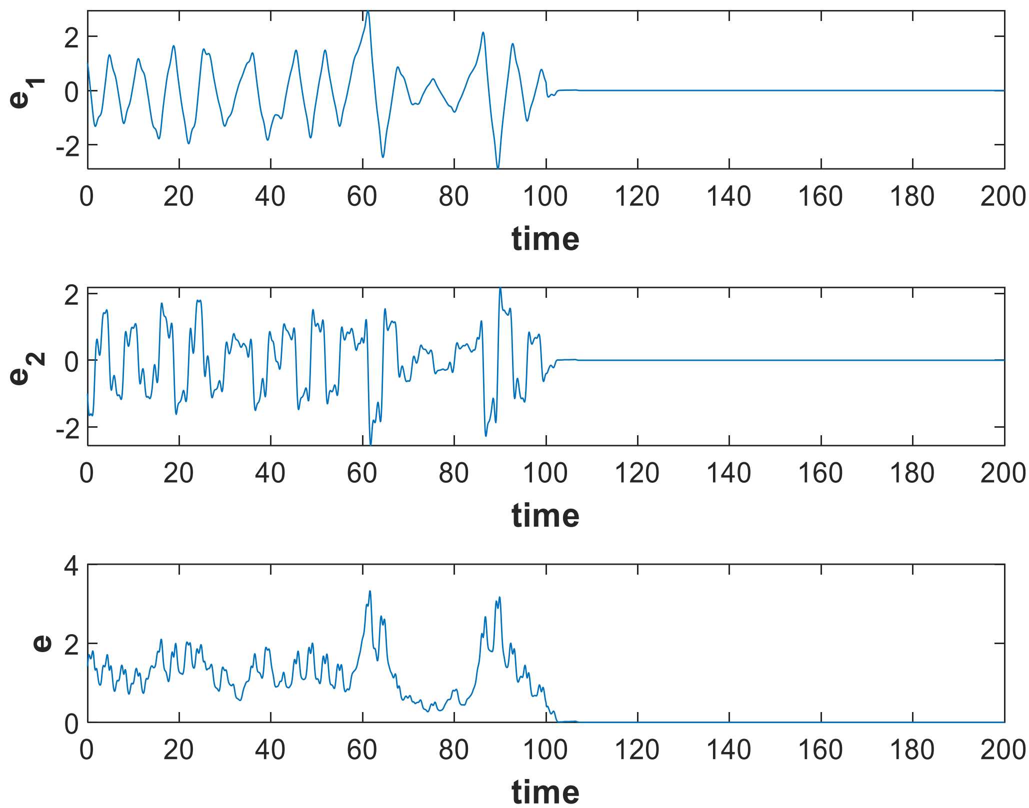

Figure 3: The dynamics of the error variables when the control functions are activated at $100 \leq t \leq 200$ where $e_1 = x_1 - y_1$, $e_2 = x_2 - y_2$ and $e = \sqrt{e_1 + e_2}$

It is observed that before the application of the control functions, the dynamics of the state variables moved along different trajectories as result of differences in their initial conditions. In order to verify stable synchronization, the two system must have a common trajectory as a result control functions are activated for $100 \leq t \leq 200$. It is notice from the Figure 2 that the systems variables achieved common trajectory or identical trajectory as from $t \geq 100$. The achievement of this common trajectory shows that stable synchronization has been realized. Similarly, from Figure 3, the dynamics of the error variable shows that the error dynamics moved chaotically with time before the activation of the control functions and immediately the control function are activate for $100 \leq t \leq 200$ the error dynamic reduced and stabilizes at zero. This again confirms the realization of stable synchronization.

Determination of coupling threshold for unidirectional coupling

In this section, algebraic criterion for unidirectional coupled Josephson junctions with coupling parameters from two different bounded dynamical regimes G_1 and G_2

Case 1: Determination of coupling threshold using the algebraic criteria G_1

$G_1 = \left\{ |x_{1,} - y_{1,}| \leq \frac{3\pi}{4}, |x_{2,} - y_{2,}| \leq \frac{3\pi}{4} \right\}$ then we have $\frac{2\sqrt{2}}{3\pi} \leq S_1(t) \leq 1$ and $\frac{2\sqrt{2}}{3\pi} \leq S_2(t) \leq 1$

In the case of unidirectional coupling the controller is chosen as $u(t) = (k_1 \sin(x_1 - y_1), 0)^T$, the sufficient criteria can then be obtained by substituting G_1 into the second inequality in Equation (14) with $k_2 = 0$ gives

$$4P_1P_2k_1\left(\frac{2\sqrt{2}}{3\pi}\right)\delta > \left(P_1 + P_2q(t)\right)^2$$

$$k_1 > \frac{3\pi\left(P_1 + P_2(1+|\xi|)\right)^2}{8\sqrt{2}\,P_1P_2\delta} \qquad (19)$$

In order to obtain sharper synchronization criteria $P = diag\{(1+|\xi|)P_2, P_2\}$ in (17) yields

$$k_1 > \frac{3\pi(1+|\xi|)}{2\sqrt{2}\,\delta} \qquad (20)$$

Case 2: Determination of coupling threshold using the algebraic criteria G_2

$G_2 = \left\{ |x_{1,} - y_{1,}| \leq \frac{\pi}{2}, |x_{2,} - y_{2,}| \leq \frac{3\pi}{4} \right\}$. Then we have $\frac{2}{\pi} \leq S_1(t) \leq 1$ and $\frac{2\sqrt{2}}{3\pi} \leq S_2(t) \leq 1$

Similarly, in the case of unidirectional coupling the controller is chosen as $u(t) = (k_1 \sin(x_1 - y_1), 0)^T$, the sufficient criteria can then be obtained by substituting G_2 into the second inequality in Equation (14) with $k_2 = 0$ gives

$$4P_1\, P_2\left(\frac{2}{\pi}\right)\delta k_1 > \left(P_1 + P_2 q(t)\right)^2$$

$$k_1 > \frac{\pi\left(P_1 + P_2(1+|\xi|)\right)^2}{8P_1\, P_2\delta} \qquad (21)$$

Now using $P = diag\{(1+|\xi|)P_2, P_2\}$ in (24), we obtain

$$k_1 > \frac{\pi}{2\,\delta}(1+|\xi|) \qquad (22)$$

Please note that numerical simulation result is only presented for case 2 since the result are similar. Now the numerical solution Equations (4) and (5) with activation of controllers defined in Equation (6) numerically using condition in equation (21) and (22) for $100 \leq t \leq 200$ using inbuilt ODE45 solver in MATLAB. The results obtained is as shown in Figure (4) and Figure (5).

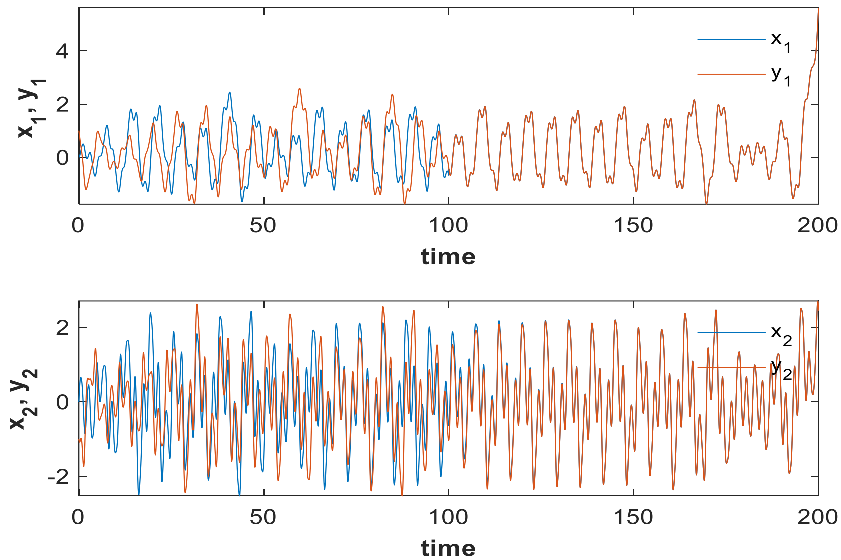

Figure 4: The dynamics of the state variables of the drive and response variable when the control functions are activated for $100 \leq t \leq 200$

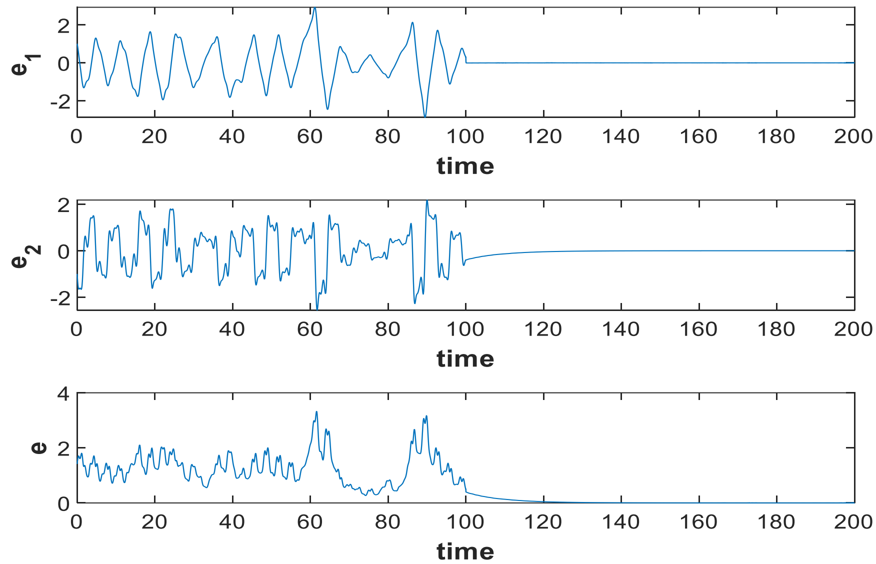

Figure 5: The dynamics of the error variables when the control functions are activated for $100 \leq t \leq 200$ where $e_1 = x_1 - y_1$, $e_2 = x_2 - y_2$ and $e = \sqrt{e_1 + e_2}$

The dynamics of the variables of two identical periodically modulated Josephson junctions evolving from different initial conditions is numerically simulated with control function within the bound region G_2 to confirm the feasibility of the analytical result. In the simulation techniques, the bounded control function was applies for$100 \leq t \leq 200$ to observe the effect of the control function on the systems. The numerical simulations result show that the dynamics of the drive and response variables followed different trajectories before the application of control function. Immediate the control function was activated, the drive and the response systems achieved identical trajectory which indicate synchronization s shown in Figure 4. Similarly, in Figure 5, it seen that the error dynamics moved chaotically with the time before the activation of control function. So, when control function was activated for $100 \leq t \leq 200$ t, the error dynamics reduce to zero which again indicate stable synchronization.

CONCLUSION

This research paper examines the synchronization of sinusoidally coupled periodically modulated Josephson junctions using Sylvester criterion. It is observed that irrespective of the dynamical region of the bounded coupling parameter, synchronization is still achievable. The sinusoidal coupling technique applied in different dynamical regimes give different ways to optimize our coupling threshold choices. Numerical Simulations presented confirm the sufficiency of the Sylvester's criterion for stable synchronization. which is a sufficient synchronization criterion for different coupling thresholds. Search is still ongoing to discover sharper analytical synchronization threshold that would give the exact minimum synchronization threshold for lower and higher order dynamical systems.

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