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Comparative Study on Electron Stopping Power in Some Material Targets Using Einstein Relativistic Mass-Energy Theory, Bahjat Mass-Energy Concept and New Mass-Energy Concept

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ABSTRACT
This study presents calculated results of the stopping power of electrons of kinetic energy, ranges from 0.01MeV to 1000MeV in some Material targets. The method employed Einstein mass-energy theory(mc^2), Bahjat mass-energy concept(mbc) and new mass- energy concept(mvc) into the Beth-Bloch radiative and collision stopping power formula for calculations of total electron stopping power in air, tissue, water, skeletal muscle, plastic, copper and lead. The Einstein mass-energy theory has been under estimated in electron stopping power. The graphical plots of stopping power using Einstein(mc^2), Bahjat(mbc)and our new mass-energy concept(mvc) verses energy show that, the rate of energy lost using mvc ride between the mc^2 and mbc; also close to mc^2compared to mbc. The curves are hyperbolic at lower electron energies and at higher energy values approximate to straight lines. The resultant particle speed constant v, describing the electron's speed, is more realistic since the electron has mass, hence moves slower than the photon (light). This work is applicable in the areas of nuclear and particle physics for the interpretations of atomic structures, calculation of nuclear binding energy and nuclear reaction energies. In the historical development of E = mc^2, it is interesting to note that this concept has been evolving continuously. If the concept of Einstein's mass-energy relationship has evolved into many areas of applications, many alternative conceptions would be formed during the last 100 years. This paper has demonstrated some of the controversies surrounding the conceptual development of E = mc^2 and there is the need to pay attention to its inclusion in any curriculum.

Keywords:
Stopping Power,
Einstein(mc^2),
Bahjat (mbc),
New (mvc),
Electron.

INTRODUCTION

The stopping power is also known as the rate of energy loss per unit length. It is determined by the velocity and charge of the falling particles as well as the target material. Furthermore, determination of the stopping power in rapid ions in metals is critical for a variety of charged particle beam applications, including material characterization and modification. In more recent, modeling metal nanoparticle radiosensitisation in ion beam is applicable in cancer therapy (Taghreed and Firas, 2019; Isabel *et al.*, 2021). The ideology of the stopping power, straggling range, and corresponding dose rate of ions in air, tissue and polymers are crucial in many researches and function in nuclear physics and radiation dosimetry (El-Ghoshal, 2017). For years, the rate of energy lost by a charged particle when it passes through matter has been worrisome (Ahlam and Khalda, 2022). Currently, the experimental and theoretical studies of the energy lost and range of charged particles have been increasing. Several theoretical and experimental studies have been conducted (Ahmed *et al.*, 2020). The charged particles lose energy as an outcome of continuous collision (Anthony *et al.*, 2017). The collision and radiative, are two component of the electron stopping power. These two Components are crucial. First, the methods employed for evaluating the two components are much different. Secondly, the energy used to ionize and excite atoms is absorbed in the medium relatively close to the electron track, while the majority of the energy lost as bremsstrahlung travels far away from the pathway before being absorbed (Berger *et al.*, 1993).The Calculations of stopping power are mostly based on the ions' effective charge and velocity (Dore *et al.*, 2014).

Theory

Bethe-Bloch Model for the Electron Stopping Power

The Bethe-Bloch formula has the following full expression:

$$-\frac{dE}{dx} = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{4\pi Z^2 N_A Z\rho}{mc^2\beta^2 A}\left[ln\left(\frac{2mc^2\beta^2}{I}\right) - ln(1-\beta^2) - \beta^2\right] \quad (1)$$

The parameter $-dE/dx$ is known as the stopping power, the negative sign indicate loss of energy, e is the electron charge, m is the mass of the electron, c is the speed of light, N_A is the Avogadro's constant, (Z/A) is the ratio of atomic number to the atomic mass, **ρ** is the density, β is the velocity of the particle relative to the speed of light and I is the excitation energy. The beth-bloch expression given in equation (1) was resolved into equation (2) and (3) for the case of electron(El-Ghoshal, 2017) .

Collision and Radiative Stopping Powers:

The average energy loss per unit path length due to inelastic Coulomb collisions with bound atomic electrons in the medium is known as collision stopping power and it was expressed as follow (Ngari and Ngadda, 2023).

$$\left(\frac{dE}{dx}\right)_c = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\left[\frac{2\pi N_0 Z\rho}{mc^2\beta^2 A}\right]\left[\frac{T(T+mc^2)\beta^2}{2I^2mc^2} + (1-\beta^2) - \left(2\sqrt{1-\beta^2} - 1 + \beta^2\right)ln2 - \frac{1}{2}\left(1-\sqrt{1-\beta^2}\right)^2\right] \quad (2)$$

Radiative Stopping Power:

The average energy loss per unit path length due to bremsstrahlung emission in the electric field of the atomic nucleus and of the atomic electron is known as radiative stopping power and it was expressed as:

$$\left(\frac{dE}{dx}\right)_r = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\left[\frac{N_0 Z^2\rho(T+mc^2)}{137m^2c^4A}\right]\left[4ln\frac{2(T+mc^2)}{mc^2} - \frac{4}{3}\right] \quad (3)$$

The sum of the collision and radiative stopping powers is known as electron total stopping power and it was expressed as following equation

$$\left(\frac{dE}{dx}\right)_T = \left(\frac{dE}{dx}\right)_C + \left(\frac{dE}{dx}\right)_r \quad (4)$$

The Einstein relativistic mass-energy theory from the Newtonian Point of view:

The concepts of Newton's law of motion including kinetic energy and potential energy were deal by Newtonian mechanics. The Einstein's relativistic mass-energy theory $E = mc^2$ was derived using this approach (Annamalai and Anthonio, 2023). Mass in general consist of different kinds of matter associated with volume. Subsequently, light is not considered as matter and it has a speed which is approximately $3 \times 10^8 metre\ per\ second$.As light has no mass and volume, the speed light is more than the speed matter (Annamalai, 2022).

The Einstein mass-energy theory denotes relativistic energy in the theory of special relativity (Annamalai, 2023a; Annamalai, 2023g; Annamalai, 2023h).

The relativistic momentum was concerned with the motion of a particle whose velocity approaches the velocity of light (Perez and Ribisi, 2022).

Rest Mass-energy equivalence ($E = m_0c^2$) from the Newtonian Mechanics :

According to Newton's second law of motion, the force (F) acting on a particle is equal to

Change of its momentum (p) with respect to time (t) and it was expressed as follows.

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dv}{dt} \text{ (Annamalai, 2023a)} \quad (5)$$

Since the kinetic is the work done by a particle, then following equation was obtained:

$$dK = dW = Fds \quad (6)$$

Substituting equation (1) for F into equation (2) the following equation was obtained

$$dK = Fds = \left(m\frac{dv}{dt} + v\frac{dv}{dt}\right)ds \quad (7)$$

The derivative of equation (3) were taken with respect to time (t)

$$dK = Fds = m\frac{ds}{dt}dv + v\frac{ds}{dt}dv \quad (8)$$

$$where, \frac{ds}{dt} = v$$

Base on hypothesis of variable mass c^2dm is actually occurs at high speed. Therefore kinetic energy is equal to c^2dm.

$$dK = mvdv + v^2dm \quad (9)$$

$$where, dK = Kinetic\ energy$$

$$c^2dm = mvdv + v^2dm \quad (10)$$

Making $\frac{dm}{m}$, the subject of relation in equation (6) the following equation was obtained:

$$\frac{dm}{m} = \frac{v}{c^2-v^2}dv \quad (11)$$

The integral of equation (7) was taken as shown in the following equation:

$$\int_{m_0}^{m}\frac{dm}{m} = \int_0^v \frac{v}{c^2-v^2}dv \quad (12)$$

$$[ln(m)]_{m_0}^{m} = -\frac{1}{2}[(c^2-v^2)]_0^v \quad (13)$$

$$ln\ m - ln\ m_0 = -\frac{1}{2}(c^2-v^2) + \frac{1}{2}lnc^2 \quad (14)$$

$$ln\frac{m}{m_0} = \frac{1}{2}ln\frac{c^2}{c^2-v^2} \quad (15)$$

$$\frac{m}{m_0} = \sqrt{\frac{c^2}{c^2-v^2}} \quad (16)$$

$$\frac{m}{m_0} = \sqrt{\frac{1}{1-\frac{v^2}{c^2}}} \quad (17)$$

$$m = \frac{m_0}{\sqrt{\frac{1}{1-\frac{v^2}{c^2}}}} \quad (18)$$

Where m symbolized the relativistic mass of the particle, m_o, the rest mass of the particle, v, the velocity of the particle and c, the speed of light$t$ (Annamalai, 2023g).

The momentum:

$$P = mv \quad (19)$$

Equation (18) were substituted into equation (19)

$$P = \frac{m_0 v}{\sqrt{\frac{1}{1-\frac{v^2}{c^2}}}} \quad (20)$$

Since the relativistic energy was given as $(E = mc^2)$ Equation (18) becomes:

$$E = \frac{m_0 c^2}{\sqrt{\frac{1}{1-\frac{v^2}{c^2}}}} \quad (21)$$

The relationship between the relativistic energy and the relativistic momentum were given as follow:

$$E^2 = \frac{m_0^2 c^4}{\frac{1}{1-\frac{v^2}{c^2}}} \quad (22)$$

$$E^2 = \frac{m_0^2 c^2 (v^2 - v^2 + c^2)}{1-\frac{v^2}{c^2}} \quad (23)$$

$$E^2 = \frac{m_0^2 c^2 v^2 - m_0^2 c^2 v^2 + m_0^2 c^4}{1-\frac{v^2}{c^2}} \quad (24)$$

$$E^2 = \left(\frac{m_0 v}{\sqrt{\frac{1}{1-\frac{v^2}{c^2}}}}\right)^2 c^2 + \frac{m_0^2 c^2 (c^2 - v^2)}{\frac{c^2 - v^2}{c^2}} \quad (25)$$

From the expression above, equation (25), the energy-momentum relation were obtained

$$E^2 = P^2 c^2 + m_0^2 c^4 \quad (26)$$

If particle is at rest, then the momentum $P = 0$ thus, the rest energy becomes $E = m_o c^2$

The Relativistic Mass-energy equivalence($E = mc^2$) was derived by employing equation (18) above

$$m = \frac{m_0}{\sqrt{\frac{1}{1-\frac{v^2}{c^2}}}}$$

On squaring both side of the equation the following were obtained

$$m^2 = \frac{m_0^2}{\frac{1}{1-\frac{v^2}{c^2}}} \quad (27)$$

$$m^2 (c^2 - v^2) = m_0^2 c^2 \quad (28)$$

Where, $m_0^2 c^2$ is the rest mass energy of the particle

$$m^2 c^2 - m^2 v^2 = m_0^2 c^2 \quad (29)$$

By differentiating the equation (29) with respect to time the following equation was obtained

$$2mc^2 \frac{dm}{dt} - 2mv \frac{d(mv)}{dt} = 0 \quad (30)$$

From equation (30), the following equation was obtained:

$$c^2 \frac{dm}{dt} = v \frac{d(mv)}{dt} \quad (31)$$

$$\frac{dE}{dt} = Fv = v \frac{d(mv)}{dt} = c^2 \frac{dm}{dt} \quad (32)$$

$$dE = c^2 dm \quad (33)$$

The kinetic energy of the particle K was given as follow:

$$\int_0^K dE = \int_{m_0}^m c^2 dm \quad (34)$$

$$K = c^2 (m - m_0) \quad (35)$$

The sum of the kinetic energy and the rest mass-energy $m_0 c^2$ is the total energy of the particle and expressed as follow:

Total Energy (E) = Kinetic Energy (K) + Rest Mass-Energy $m_0 c^2$

$$E = c^2 (m - m_0) + m_0 c^2 \quad (36)$$

Hence

$E = mc^2$ (Perez and Ribisi, 2022). (37)

Bahjat Mass-Energy concept(mbc):

Bahjat reported that, the Einstein's relativistic mass–energy theory $E = mc^2$ overestimates the nuclear energy. Therefore, the relativistic formula was suggested with another energy conversion factor (bc), other than (c^2) to help in the computation nuclear energy data (Bahjat, 2008). Therefore, the energy converting factor by Bahjat *bc was used to replace* c^2 in Einstein relativistic mass-energy $E = mc^2$ to have the following equation

$$E = mbc. \quad (38)$$

Where:

$E =$ *is the energy of electron*

$m =$ *is the mass of electron*

$b =$ *is the energy converting factor given as* $0.6037970064 \times 10^8 m/s$ (Bahjat, 2008).

$c =$ *is the velocity of light in vacuum given as* $3 \times 10^8 m/s$.

MATERIALS AND METHODS

Firstly, the method looked at Einstein mass-mass energy converting factor (c) and Bahjat mass-energy converting factor (b) as given in equation (37) and (38) to deriving new mass-energy converting factor (v) as $1.8 \times 10^8 m/s$ through the application of Energy-frequency relation($(E = h/\lambda)$), Blackbody Energy-temperature relation($E = KT_{Blacbody}$), Wein's displacement relation($\lambda = b/T_{Blackbody}$), Particle momentum relation($P = mc$), and velocity-wavelength relation($c = f\lambda$)used to obtained new mass-energy concept ($E = mvc$).The Bahjat mass-energy relation(mbc) and the new mass-energy concept(mvc) were employ into electron collision and radiative stopping power given in equation (2) and (3) to replace Einstein relativistic mass-energy theory(mc^2)as in the following equations:

Using Bahjat mass-energy relation (mbc), equation (2) and (3) is transformed to:

$$\left(\frac{dE}{dx}\right)_C = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \left[\frac{2\pi N_0 Z\rho}{mbc\beta^2 A}\right]\left[\frac{T(T+mbc)\beta^2}{2I^2 mbc} + (1-\beta^2) - \left(2\sqrt{1-\beta^2} - 1 + \beta^2\right)ln2 - \frac{1}{2}\left(1-\sqrt{1-\beta^2}\right)^2\right] \quad (39)$$

$$\left(\frac{dE}{dx}\right)_r = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \left[\frac{N_0 Z^2\rho(T+mbc)}{137m^2b^2c^2A}\right]\left[4ln\frac{2(T+mbc)}{mbc} - \frac{4}{3}\right] \quad (40)$$

Where $\beta = \frac{2E}{mbc} = \frac{V^2}{bc}$

Using new mass-energy concept(mvc), equation (2) and (3) is transformed to:

$$\left(\frac{dE}{dx}\right)_C = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \left[\frac{2\pi N_0 Z\rho}{mvc\beta^2 A}\right]\left[\frac{T(T+mvc)\beta^2}{2I^2 mvc} + (1-\beta^2) - \left(2\sqrt{1-\beta^2} - 1 + \beta^2\right)ln2 - \frac{1}{2}\left(1-\sqrt{1-\beta^2}\right)^2\right] \quad (41)$$

$$\left(\frac{dE}{dx}\right)_r = \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \left[\frac{N_0 Z^2\rho(T+mvc)}{137m^2v^2c^2A}\right]\left[4ln\frac{2(T+mvc)}{mvc} - \frac{4}{3}\right] \quad (42)$$

Where $\beta = \frac{2E}{mvc} = \frac{V^2}{vc}$

To compute the stopping power of electron, we first calculate the relativistic energy of electron (mc^2), velocity of electron relative to speed of light ($v^2/c^2 = 2E/mc^2 = \beta^2$) and electron density (n $=N_0\rho\frac{Z}{A}$) which employ collision and radiative stopping power formula as given in equation (2) and (3) by the use of computer excel program within the energy range 0.01MeV to 1000MeV. Secondly, we compute the Bahjat mass-energy (mbc), electron velocity relative to the product of mass energy converting factor (b) and speed of light ($v^2/bc = 2E/mbc = \beta^2$)which employ collision and radiative stopping power formula as given in equation (39) and (40). Thirdly, we compute the new mass-energy concept (mvc),electron velocity relative to the product of mass energy converting factor (v) and speed of light ($v^2/vc = 2E/mvc = \beta^2$)which employ collision and radiative stopping power formula as given in equation (41) and (42)and electron density. The electron stopping power fitting data of Table 1 is used.

Table 1: Calculated electron density (n) in air, tissue,water,skeletal muscle, plastic, copper and lead

Material	I (eV)	Density ρ (g/m³)	Z/A	n =Nρ Z/A (electron/m³)
Air	85.7	0.00129	0.499	3.63×10^{20}
Tissue	63.2	0.92	0.558	3.09×10^{29}
Water	75	1.00	0.555	3.34×10^{29}
Skeleton	75.3	1.04	0.549	3.44×10^{29}
Plastic	65.1	1.127	0.549	3.73×10^{23}
Copper	322	8.960	0.456	2.46×10^{24}
Lead	823	11.35	0.396	2.70×10^{24}

RESULTS AND DISCUSSION

Table 1 Depicted the calculated electron density (n) of water, tissue, skeletal muscle, air, plastic, copper and lead. Observation depicted that the materials are listed in order of increasing densities: air, tissue, water, skeletal muscle, plastic, copper and lead. However, in terms of electron density per unit volume the order is almost inverted: air, plastic, copper, lead, tissue, water, skeletal muscle, in increasing order while atomic number per unit mass decreases from tissue down to lead, while the excitation energies are extremely high for metallic copper and lead. Figs. 1 to 7 below depicted the graphical extrapolations of the total electron stopping power verses energy. Correlations between the calculated electron stopping power using mass-energy relations; $mc^2, mbc\ and\ mvc$ are presented. The graphical extrapolations depicted that, the electron total stopping power verses energy using mvc, is close to mc^2 compare to mbc, also rides between mc^2 and mbc for: air; tissue; water; skeletal muscle; plastic. The curves are hyperbolic at lower electron energies and at higher energy values approximate to straight lines.

The electron stopping power using our new mass-energy concept (mvc) gives an average energy lost compared with that of Bahjat (mbc) and it ride between Einstein (mc^2) and Bahjat(mbc). Since electron is relatively massive in nature, it energy converting factor v, is more realistic compared to c which is only attributed to light or photon.

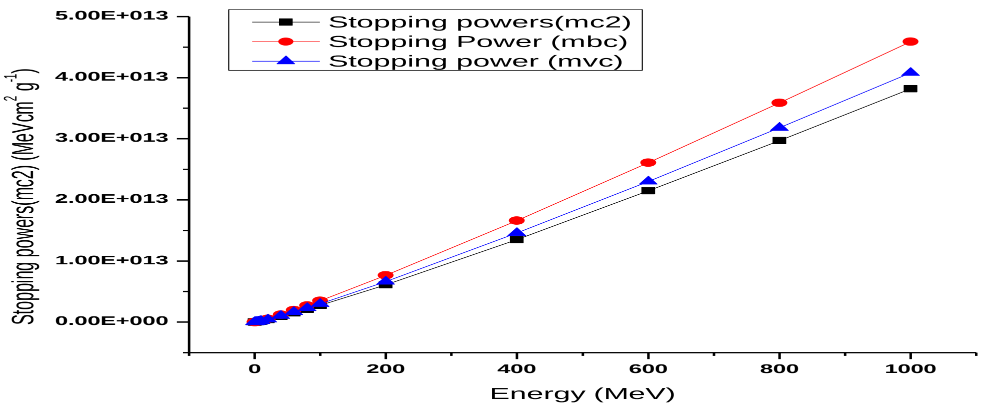

Figure 1: Graphical Presentation of electron stopping power in water

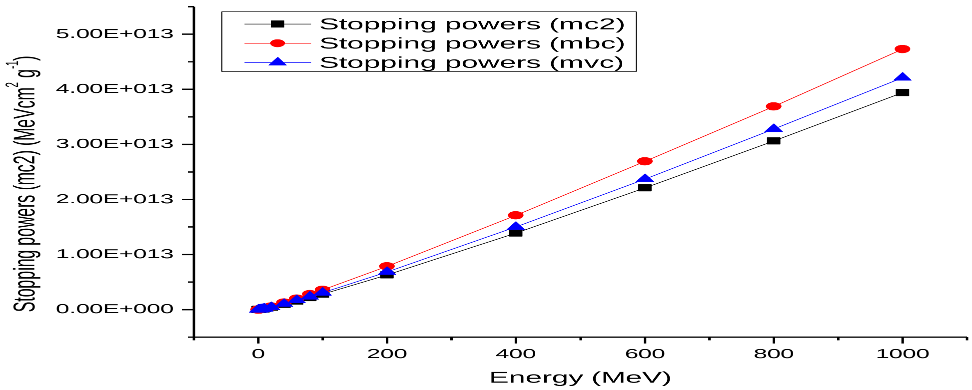

Figure 2: Graphical Presentation of electron stopping power in tissues

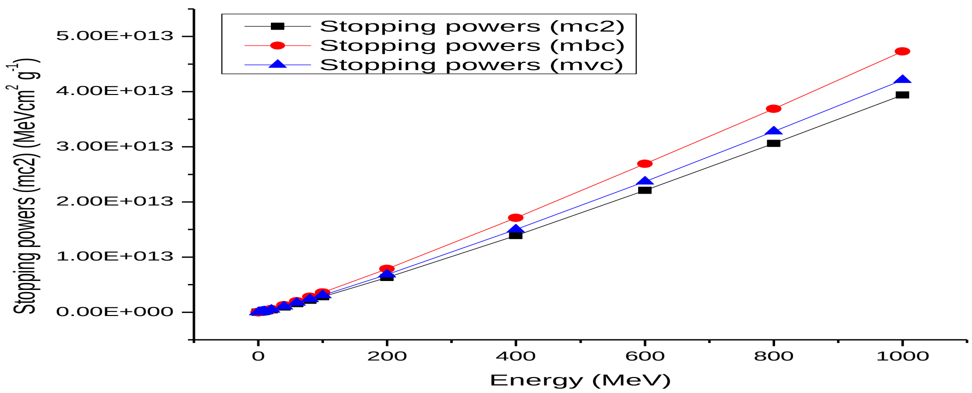

Figure 3: Graphical Presentation of electron stopping power in skeletal

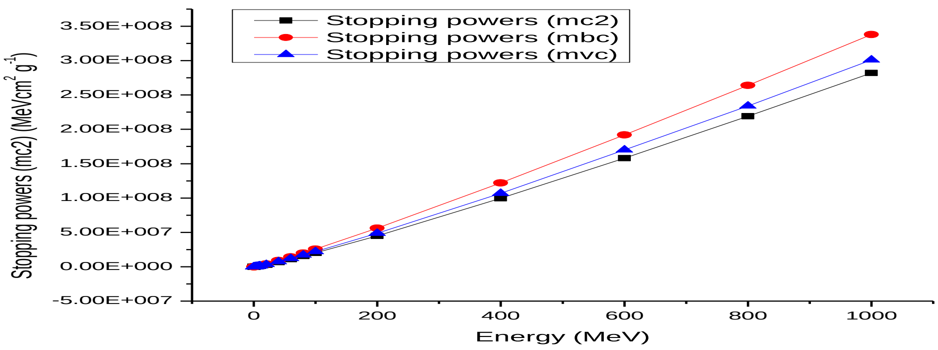

Figure 4: Graphical Presentation of electron stopping power in copper

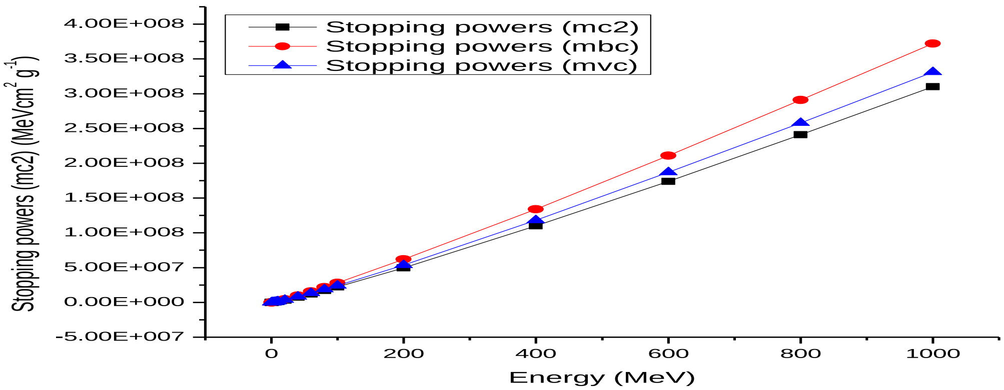

Figure 5: Graphical Presentation of electron stopping power in lead

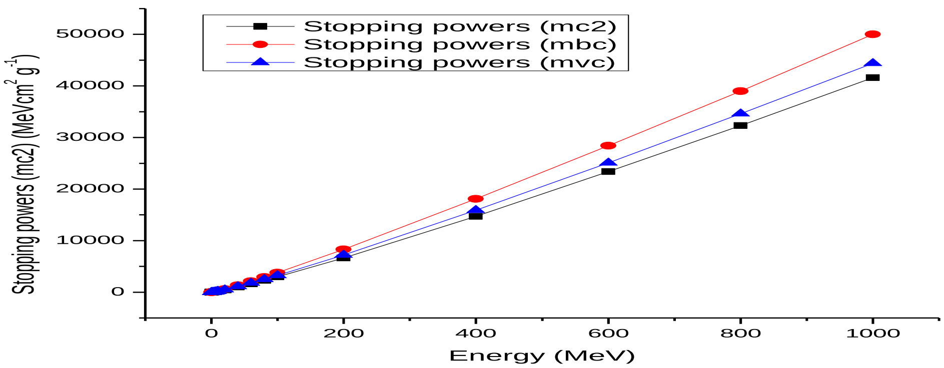

Figure 6: Graphical Presentation of electron stopping power in Air

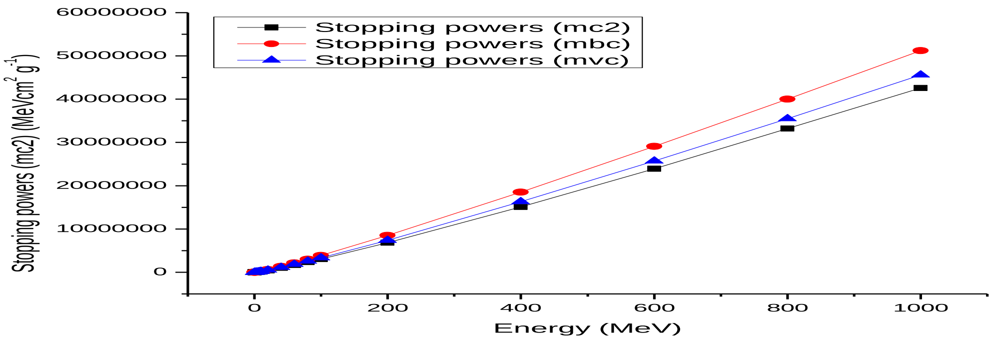

Figure 7: Graphical Presentation of electron stopping power in Plastic

CONCLUSION

The new mass-energy concept, mvc, has been incorporated into the Beth-bloch stopping power formula in computation of the total electron stopping power in some materials target namely, air, tissue, water, skeletal muscle, plastic, copper and lead within the electron energy range of 0.01MeV to 1000MeV.

The new mass-energy concept mvc is approximately to mc^2 at lower energy, and close to mc^2 at medium and higher energy compare to mbc also rides between mc^2 and mbc for: air; tissue; water; skeletal muscle; copper; lead and plastic. The derived particle velocity v, describing the velocity of the electron, is more realistic since electron move slower than speed of light. In the light of that, the new mass-energy concept mvc will help in measuring the masses of the various particles in low and high energy states to replace the relativistic mass-energy mc^2which results in various applications in nuclear physics and chemistry.

For centaury, it is important note that, the Einstein's relativistic mass-energy theory mc^2 has been evolving continuously in many fields and application in nuclear and particles physic, alternative conceptions would be formed during this past years. This paper has established some of the controversies immediate the conceptual development of E = mc^2 and there is the need to pay attention to its inclusion in any prospectus.

The new mass – energy concept mvc can be employ to the areas of nuclear and particle physics, nuclear chemistry for investigations interpretations of atomic structures, calculation of binding energy per nucleon and the energy release in nuclear reactions.

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