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Determination of the η Parameter as function of Neutrino Mass: A Theoretical Approach

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ABSTRACT

Computation of the neutrino masses and η parameter experimentally abound in literature, while little or no attention has been made to determine it theoretically. However, a recent theoretical study sought to determine the masses of the neutrino, but failed to compute the value for the η parameter. Hence, this study was aimed at determining theoretically the η parameter from neutrino masses addition as predicted with quantum gravitational couplings/effective Majorana dimensionless coupling via spherical symmetry vacuum solution. A seesaw mechanism (ala mode matrix) was adopted; where the SU matrix was diagonalized to get the mass eigen states. The η parameter value which is expressed as a function of the three mass eigen states of the neutrino masses, and which also satisfies the experimental constraints was determined theoretically to be 0.06.

Keywords:
Neutrino mass,
Seesaw mechanism,
Eigen states,
SU matrix.

INTRODUCTION

Neutrinos are elusive subatomic particles created in a wide variety of nuclear processes. Their name, which means "little neutral one," refers to the fact that they carry no electrical charge (Kaneyuki and Scholberg, 1999). Of the four fundamental forces in the universe, neutrinos only interact with two — gravity and the weak force, which is responsible for the radioactive decay of atoms (Hirsch et al., 2013). Having nearly no mass, they zip through the cosmos at almost the speed of light. They're also extremely common—in fact, they're the most abundant massive particle in the universe (Castelvecchi, 2019). Neutrinos come from all kinds of different sources and are often the product of heavy particles turning into lighter ones, a process called "decay" (Lipari, 2003).

Natural sources of neutrinos include the radioactive decay of primordial elements within the earth, which generate a large flux of low-energy electron-anti-neutrinos (Leyton et al., 2017). Calculations show that about 2 percent of the sun's energy is carried away by neutrinos produced in fusion reactions there (Orebi Gann et al., 2021). Supernovae too are predominantly a neutrino phenomenon, because neutrinos are the only particles that can penetrate the very dense material produced in a collapsing star; only a small fraction of the available energy is converted to light (Bethe and Brown, 1985). It is possible that a large fraction of the dark matter of the universe consists of primordial, Big Bang neutrinos.

Although, precise masses m_1, m_2 and m_3 of neutrinos and the ordering of these masses are currently poorly determined features of particle physics. Solar, atmospheric, accelerator and reactor neutrino experiments are able to determine mass-squared differences, and the so called Mikheyev-Smirnov-Wolfenstein matter effect determines the ordering of two of the mass eigenstates (Langacker, 2011).

For instance, Vagnozzi et al. (2017) unveiled neutrino secrets with cosmological data: neutrino masses and mass hierarchy; where they assumed flat Lambda CDM cosmology. Here, Strongest bound on the sum of the three masses of neutrino was derived.

Vagnozzi et al. (2017) while revolving around the dynamical dark energy models in surveying cosmological limitations on the addition of the three neutrino masses in the context of dynamical dark energy (DDE) models with equation of states (EoS), concluded that the constraints on the addition of the masses do not degrade with respect to those obtained in the CDM; they slightly tighter despite the enlarged parameter space. Heavens and Sellentin (2018), using the constructed uninformative pior from principles of the Objective Bayesian approach found that the normal hierarchy of the neutrino is favoured but with inconclusive posterior odds.

Also, the Majorana neutrino mass matrix was diagonalized by Duarah (2019) using PMNS matrix; analytical relations between the mass matrix elements and mixing parameters were obtained, viz., three mixing angles - θ_{12}, θ_{23}, θ_{13} and Dirac CP phase \delta. His results arising from special μ-τ symmetric mass matrix corresponds to maximal atmospheric mixing ($\theta_{23}=\pi/4$) and maximal CP violation ($\delta=-\pi/2$), showing a deviation of θ_{23} from its maximal value which can be correlated with the prediction of other two mixing angles. Massimiliano et al. (2020) extended the standard cosmological model with massive masses. Recent cosmological data and neutrino oscillation experiments highly limits Quasi-degenerate neutrinos. However, without reference to cosmological information, the absolute masses are currently relatively poorly limited, with different 90% credible upper limits for m_1 in the range $0.18 - 0.48$ eV, relying on the datasets used. This creates possibilities of m_3 being larger than m_1 and m_2 (referred to as normal hierarchy, NH), and smaller than m_1 and m_2 (referred to as the inverted hierarchy; IH), still for the first time in literature, they did justice to the experimental value η parameter for a full-fledged scenario.

Though many researchers have failed to calculate theoretically the sum of the neutrino masses until Olakanmi and Farida (2021), where from the Bose-Einstein statistical modification to gravitation, spherically symmetric vacuum solutions were obtained from the quantum gravitational couplings/effective Majorana dimensionless couplings. However, they failed to determine the η - parameter. Therefore, in this paper, we investigate the applicability of theoretical values calculated by Olakanmi and Farida (2021) using the seesaw mechanism to obtain η-parameter which has not been obtained theoretically.

MATERIALS AND METHOD

Seesaw Mechanism

According to Klauber (2013), m_v which is the mass matrix of the neutrino, takes the form

$$m_v = \begin{pmatrix} 1+2a_2+a_3 & a_4 & a_4 \\ a_4 & a_2 & a_1+a_2 \\ a_4 & a_1+a_2 & a_2 \end{pmatrix} \quad (1)$$

where the η parameter ≡ $1 + 2a_2 + a_3$, particularly in full-fledged view. It should be noted that the normal ordering where m_3 is greater than m_1 and m_2 is realized for magnitude of the η parameter lesser than 1, while inverted ordering where m_3 is lesser than m_1 and m_2 is realized for η parameter greater than 1.

Sum of the individual masses $\sum m_v$ and η are related by the formula

$$\sum m_v = \left[\frac{\Delta m_{31}^2}{1-\eta^2}\right]^{\frac{1}{2}} (1+2\eta) \qquad (2)$$

Where $\eta < 1\ and\ \eta > 1$

The generalized relationship between the addition of the masses $\sum m_v$ and η for every $\eta' > \eta$ is given by:

$$\sum m_v = \left[\frac{\Delta m_{31}^2}{1-[\eta-\eta'(\eta)]^2}\right]^{\frac{1}{2}} (2\eta+1) \qquad (3)$$

RESULTS AND DISCUSSION

Eigen Values

Diagonalising the mass matrix of the neutrinos using the Mathematical software will give the following eigen values

$$m_1 = a_1 + 2a_2 + a_3 - \left(\sqrt{{a_3}^2 + 2{a_4}^2}\right) \qquad (4)$$

$$m_2 = a_1 + 2a_2 + a_3 + \left(\sqrt{{a_3}^2 + 2{a_4}^2}\right) \qquad (5)$$

$$m_3 = -a_1 \qquad (6)$$

Where a_1, a_2, a_3, η and η^I are parameters of neutrinos

m_1= mass of the electron neutrino
m_2= mass of the moun neutrino
m_3 = mass of the tau neutrino
Δm_{31}^2 = the squared mass differences of the electron neutrino = 2.5 x 10^{-3}
$\sum m_v$ = sum of the neutrino masses

Model Equation

$$\eta = a_1 + 2a_2 + a_3 \qquad (7)$$

$$\eta^I = \left(c^2 + 2{a_4}^2\right)^{\frac{1}{2}} \qquad (8)$$

$$\Delta m_{31}^2 = {a_1}^2 - 2(a_1 + 2a_2 + a_3)^2 - 2(a_1 + 2a_2 + a_3)\left({a_3}^2 + 2{a_4}^2\right)^{\frac{1}{2}} + {a_3}^2 + 2{a_4}^2 \qquad (9)$$

$$\Delta m_{31}^2 = {a_1}^2 - (\eta)^2 - 2(\eta)(\eta^I) + (\eta^I)^2 \qquad (10)$$

$$\sum m_v = m_1 + m_2 + m_3 \qquad (11)$$

$$\sum m_v = a_1 + 2a_2 + a_3 - \left(\sqrt{{a_3}^2 + 2{a_4}^2}\right) + a_1 + 2a_2 + a_3 - \left(\sqrt{{a_3}^2 + 2{a_4}^2}\right) \qquad (12)$$

$$\sum m_v = 2\eta - a_1 \qquad (13)$$

The following equation can be established from the relations above

$$\left(\sum m_v\right)^2 = (2\eta - a_1)^2 \qquad (14)$$

$$\left(\sum m_v\right)^2 = 3\eta^2 - 4a_1 \qquad (15)$$

$$\left(\sum m_v\right)^2 = 3\eta^2 - \eta[1 - 2\eta^I(\eta)] + \eta^I(\eta) - \Delta M_{31}^2 \qquad (16)$$

$$\sum m_v = \sqrt{3\eta^2 - \eta[1 - 2\eta^I(\eta)] + \eta^I(\eta) - \Delta M_{31}^2} \qquad (17)$$

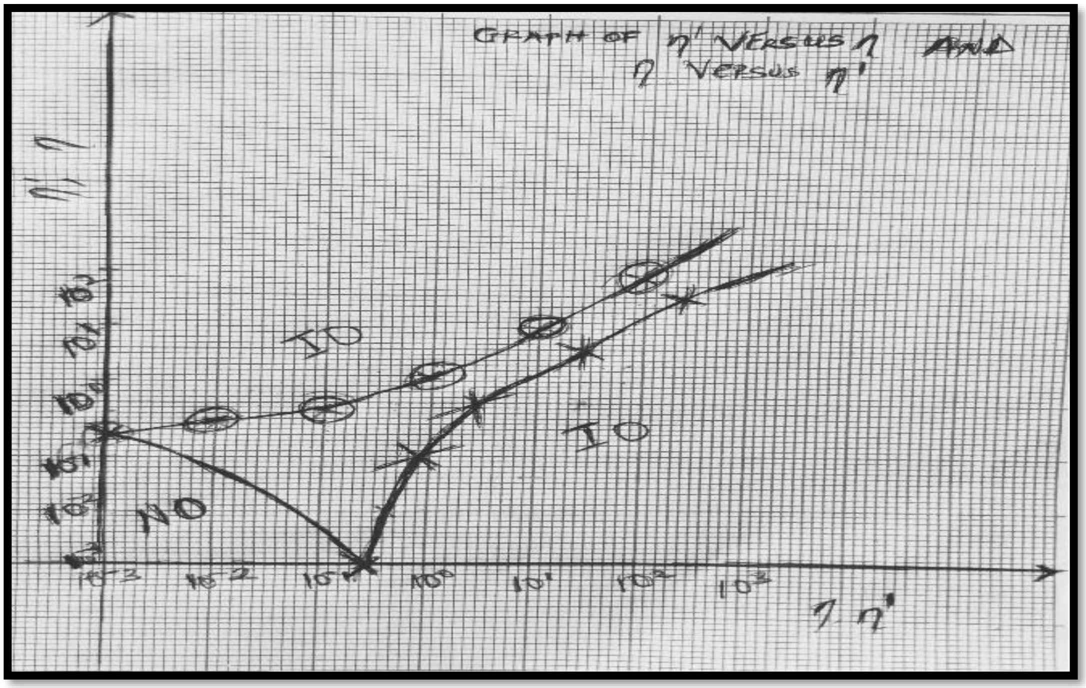

Figure 1: **η** versus **η**^{I}

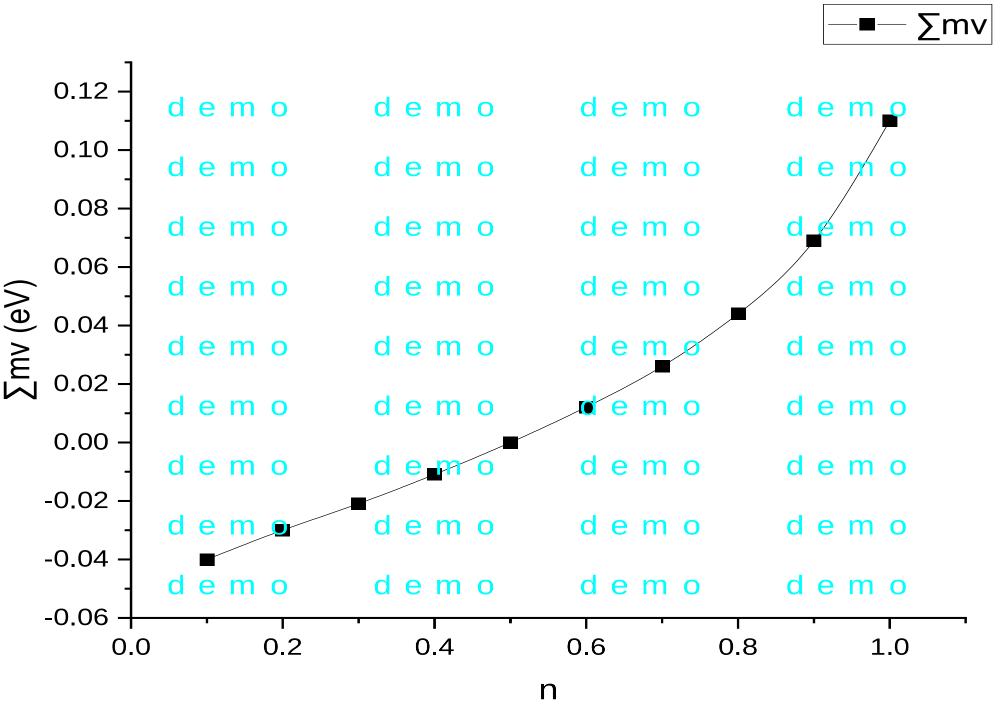

Figure 2: $\sum M_v$ versus **η**

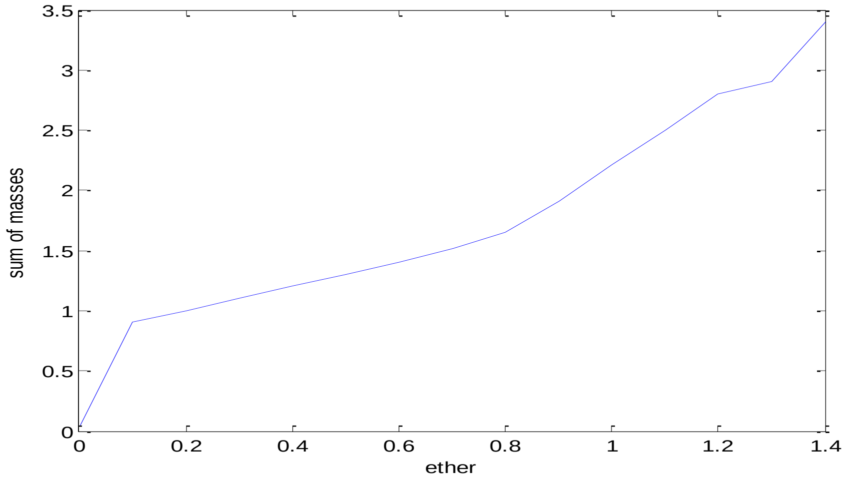

Figure 3: $\sum m_v$ versus **η** (where **η**>**η**^{I})

Figure 1 gives a plot of **η**^{I} versus **η** with **η**>**η**^{I} where normal ordering (NO) and inverted ordering (IO) are achieved. That is, where the conditions m_1 lesser than m_2, and m_2 lesser than m_3 or m_3 lesser than m_2, and m_2 lesser than m_1 holds. It is observed that just a region and two disconnected regions for NO and IO respectively satisfies the neutrino oscillation measurement in terms of $|\boldsymbol{m_3^2}| |\sum \boldsymbol{m_1^2}| |\Delta \boldsymbol{m_{31}^2}|$.

A plot of $\sum m^2$ versus **η** for the full matrix is presented in Figure 2. We assume that **η**>**η**^{I} is always possible. The graph clearly shows hierarchically the inverted ordering (NO) from experimental constraints. Note, the experiments of oscillation desire that **η**≥0.09 equivalent to $\sum m_v = 0.06eV$.

Furthermore, a plot of $\sum m_v$ versus **η** in Figure 3, assumed that **η**>**η**^{I} is always possible. The graph allows hierarchically, the inverted ordering (NO) from experimental constraints. Note, the experiments of oscillation desire that **η** ≥0.09 equivalent to $\sum M_v = 0.06eV$ which compares favourably from the calculated value from quantum gravity, i e; at $\sum m_v = 0.058eV$, **η** = 0.06, which also satisfies the constraint at **η** ≥ 0.09.

Table 1: Comparative Analysis with Previous Experimental Results

S/N	Authors	Method	Results
1	Planck collaboration 2018	Cosmic Microwave Background (Experimental)	**η** < 0.88 or **η** >1.2
	Planck collaboration 2018 + BAO	Baryon Acoustic Oscillations (Experimental)	**η** < 0.66 or **η** >1.8
2	Massimiliano et. al (2020)	Baryon Acoustic Oscillations (BAO): Experimental	**η** ≥ 0.09
3	This work	Seesaw mechanism (Theoretical)	**η** ≥ 0.06

CONCLUSION

The mass matrix, the squared mass difference, the neutrino masses summation, $\sum m_v$ and the η parameter have been diagonalised theoretically at $\sum m_v = 0.058$ as η ≥0.06 which satisfies the experimental constraints.

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